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Supplement to the Annual Report

ON THE HEALTH OF

The Urban Sanitary District of Haydock,

FOR THE YEAR 1897,

CONTAINING—

(1) A LIFE-TABLE for Haydock, based on the Mortality of the 10 years, 1881—90,

AND

(2) Other Tables, setting forth the leading Vital Statistics for the same Decennium.

BY

T. E. HAYWARD, M.B. (LOND.)

MEDICAL OFFICER OF HEALTH.

London School of Hygiene & Tropical Medicine
6 JUN. 1929

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HAYDOCK,

3rd March, 1898.

To the Chairman of the Haydock Urban District Council.

SIR,

In addition to the usual Annual Report for the year 1897, with Appendix of Tables, I have prepared for submission to your Council, a Supplementary Report, in which an attempt has been made to so deal with the Vital Statistics of Haydock for the ten years 1881–90, as to enable them to be fairly compared with those for the whole of England and Wales given in the "Decennial Supplement" of the Registrar General. It may seem rather late to present this work, but this is the earliest year in which *all* the statistics for England and Wales to be brought into comparison were available, as some of them were not issued until towards the end of 1897.

An endeavour has been made to simplify and explain as fully as possible what may be learned from these figures, and I trust, that with but a little patient attention, your Council may find sufficient interest and value in these Statistics to make it appear worth while to have spent the considerable sum necessary to defray the cost of printing.

I have the honour to be, Sir,

Your obedient Servant,

T. E. HAYWARD, M.B. (Lond.),

Medical Officer of Health.

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SUPPLEMENT TO THE ANNUAL REPORT ON THE HEALTH OF

The Urban Sanitary District of Haydock,

FOR THE YEAR 1897.

IN addition to the Annual Reports issued by the Registrar-General, it has been customary for the Superintendent of Vital Statistics for the time being, to prepare "Decennial Supplements" relating to the respective ten-yearly periods most nearly corresponding to the intervals between two successive Censuses.

The labour involved in drawing up the elaborate Tables in these Reports, and in carefully weighing all the deductions to be drawn from them, is so great, that a considerable time must elapse after the close of each Decennium, before the Supplement dealing with it can be issued.

The Decennial Report for the ten years 1881-90, was published in two volumes—Part I in 1896, and Part II in 1897, and it is such a mine of information that it can scarcely be exhausted before the time comes for its successor to appear.

Like the preceding ones, but even in a still greater degree, this Report is not merely a summary and recapitulation of statistics already given in the several Annual Reports, but, with masterly grasp and power, it reviews the vital history of the period to which it relates, in the light of the more exact knowledge as to the true mean population for the 10 years, to be obtained by *combining* the results of the two censuses of 1881 and 1891.

It has appeared to me, as it may doubtless have appeared to many other Medical Officers of Health, that it might be worth while, even for a district so small as Haydock, to make some attempt to draw up a Decennial Supplement for the corresponding period of 1881-90. Such a Supplement will at least enable comparisons, as exact as possible, to be made between the Vital Statistics for this District, and those for England and Wales as a whole, for the period in question, while, should similar Supplements be prepared in the future, this one will give a basis of exact comparison with those to come after.

A series of Tables have therefore been prepared which have been grouped together in this Supplement to my Annual Report for 1897, which may be thus summarised.

(1) A "Life Table" with preliminary Tables of Population and Deaths.

(2) Tables showing comparision of the Death-rates for Males and Females at at different age periods with the corresponding figures for England and Wales.

(3) Tables showing the mean age and sex distribution of the Population of Haydock, for the 10 years in question, with the similar statistics for England and Wales.

(4) A Table showing the calculation for Haydock of what are called in the Registrar-General's last Decennial Report, the Death-rates in the "STANDARD POPULATION," that is, showing what the mean annual Death-rates for England and Wales *as a whole* would have been during 1881-90, if the Death-rates in the various age and sex groups of the population of England and Wales had been the same as those of Haydock during the same period, thus permitting an EXACT comparison to be made.

LIFE-TABLE.

General Remarks on the Nature, Mode of Construction, and uses of a Life-Table.

A "Life-Table" may be described as a set of figures brought together so as to form a Life-measurer—a measurer of the probable length of lives in the future—on the basis of calculations from facts relating to Lives and Deaths during some definitely defined period of the past.

Sanitary improvement, which is so much talked of, and from which so much is expected, has to be finally judged as to its effects and results, by the answers to these two questions: (1)—Are lives more healthy? (2)—Are lives any longer?

In answering the latter question a Life-table is, to those concerned with Public Health, as has been well said by Dr. Tatham, what the "two-foot rule" is to the mechanic. Short of exact measurement, impressions as to the relative size and length of objects can only be approximate, and often vague. The last and final test is the application of the "rule," it may be one divided into minute fractions of an inch, or one of feet multiplied many times.

A Life-Table, then, is an instrument of exact measurement applied to estimate the number of years which human lives may be expected on the average to last, somewhat as a Thermometer is used to measure degrees of heat or cold, or as a Barometer, to measure the weight of the atmosphere.

In order to explain to some extent, the nature and mode of construction of a Life-Table, an illustration used by Dr. Tatham, formerly Medical Officer of Health for Manchester, and now Superintendent of Vital Statistics at the General Register Office, in his introduction to the Manchester Life-Table, may be, with due acknowledgment, appropriated.

It must be imagined that some years over a century ago, an immense Ledger of a million pages was opened, and that on each page was inscribed the name of an infant with sex and exact date of birth marked. For the sake of simplicity, let us imagine these infants were all born on the first day of January. It must be further supposed that by successive, careful and diligent record-keepers, the Ledger was kept regularly posted up once a year, and that, when a year after the account was opened, the work of posting was done, in the cases of all the infants who had died during the first year of life, the *fact* and *date* of death were correctly noted, and in the cases of all the survivors, one year of life was duly credited to their respective accounts. The work of posting will have been very heavy for this first year, for while it is true, alas! all through life that—

> "Dangers stand thick through all the ground
> To push us to the tomb."

these dangers stand very thick indeed during the first year of existence.

Let it be imagined that the lighter work of the second year is completed, and that it is proceeded with year after year, until the last scattered survivors, who have lingered over a century, have one after another passed away, and the last account has been finally closed.

Now, in such a Ledger the crude materials for a Life-Table would exist. Out of the original million a certain number would have died in the first year of life, a certain number in the second year of life, and so on until the end. By adding up all the years of life lived through by the original million (including not only the *complete* years, but the *parts of years* lived by those who died in the interval from the beginning of each year, to the beginning of the next) and dividing the sum by a million, what is called the "Mean Expectation of Life at Birth" would be obtained. By further adding up the total number of years lived through by the survivors at the end of the first year, and dividing the sum by that number of survivors, the mean expectation of life at the age of one year will be obtained, and so on until the end. It should be noted, however, that, in using such a Ledger, a *separate* calculation would also be made for the male and female infants respectively.

It is obvious that the duration of life in the cases of the million who were at the beginning placed in the Ledger would have been affected by two things :—

(1) Their original stock of vitality.

(2) The nature of the surroundings amidst which they had lived, more or less favourable to lengthened existence.

Now, the practical difficulties in the way of constructing a Life-Table in the way just indicated are such as to be altogether insuperable. Even were it otherwise, and the work commenced to-day, a century and more must elapse before the results are available, and during that time the conditions of existence would probably have changed so much that these results would be unreliable.

It remains to explain how by certain calculations it is possible to construct a Life-Table which is of *even greater practical value and accuracy* than the one just supposed to be constructed by observing and recording the facts relating to a million persons during their whole life-time.

The crude materials required in order to construct a Life-Table for any district are :—

(1) The numbers of Population enumerated at two successive censuses, arranged in groups according to *sexes* and *ages.*

(2) The Records of Deaths registered in the District during the 10 calendar years nearest to the two Census Dates, classified in age and sex groups corresponding to those of the Population numbers.

The general principle is to calculate from the Population numbers and the number of Deaths for each age-group for each sex the rates of mortality, and the probabilities of survival from one age to the next, and to assume that all those now living will in turn be subjected to the same death-rates as those existing at the present time.

As will appear from the further explanations to be given, no small part of the labour involved in making out a Life-Table is concerned with compiling and calculating the *correct foundation numbers.*

It may be stated at the outset, that it is of even greater importance to secure *extreme accuracy* in the numbers of Deaths to be taken into consideration, than in the Population numbers, for it is obvious (1) that Death-rates are made higher by (*a*) either diminishing the Population number or (*b*) by increasing the number of Deaths, and *vice versâ*, and (2) that a deficiency of one in the number of Deaths will have a many times greater effect in diminishing the Death-rate than an excess of one in the Population number. Thus, with two Deaths in a Population of 100 the rate of mortality is 2 per cent. Diminish the Deaths by one, and the mortality is 1 per cent. With two Deaths in a population of 101, the mortality is 1·99 per cent.; only diminished by a very small fraction.

It may now be proceeded with to explain the way in which the Preliminary Tables of Population numbers and of Deaths have been arrived at.

The basis of the Population numbers will be necessarily the numbers enumerated at the censuses of 1881 and 1891, which are given in the Appendix to the Annual Report, and it may appear on first view, that it is only necessary to add together the numbers in the respective age and sex groups, as found at the two censuses, and divide by two, thus getting the "Arithmetical means," to obtain the working numbers.

However there are certain difficulties which successively arise, on closer consideration, which make the matter much more complicated.

I. POPULATION.

The great out-standing difficulty, in this respect, with regard to Haydock, arises from the fact that within its boundaries is situated the large private Lunatic Asylum at Haydock Lodge, the Inmates and Resident Staff of which are included in the total enumerated census population numbers. The Resident Staff may of course be included fairly among the ordinary population, but *not* the Inmates.

With regard to this circumstance there are the proverbial "three courses" open:—

(1) To take the total Population number and the total number of Deaths *including* those at the Asylum.

The great objection to this proceeding is that by so doing the total Death-rates of Haydock would be so considerably increased, and in particular at some of the age-periods, as to give a quite unfair and inaccurate representation of the true Vital Statistics, as affecting the ordinary population of the District. For the period under consideration, whereas the total mean *inmate* population of the Asylum increases the ordinary mean population of Haydock by 3 per cent, the total Inmate Deaths added to the Ordinary Deaths effect an increase 10·5 per cent, and at some of the age-periods the rate of mortality would be *more than doubled.*

(2) To take the total population numbers *including* the inmate population of the Asylum, and to leave out altogether the Deaths of the inmates of the Asylum, while including those belonging to the resident staff. By this course an error will be introduced of an opposite direction but of much less extent, that is, the Death-rates will be made to appear *lower* than they ought to be. As a matter of fact, all the statistics for Haydock hitherto presented in the ordinary Annual Reports, have been based on this fallacious mode of calculation, and, thus, the Death-rates have all been made rather lower than they should have been. In the calculations necessary for a Life-Table, however, it is of such great importance to secure the *utmost attainable accuracy* in the foundation numbers, that this second course is unsuitable if a better one can be found.

(3) There remains the last course, and the best, if it can possibly be secured, viz: to eliminate altogether from the calculation both the Population number and the number of Deaths as regards the *inmates* of the Asylum. The Resident Staff, to the number of from sixty to seventy, may of course be fairly included in the ordinary Population of the District, and the Deaths occurring among them may also be included among the Deaths belonging to Haydock proper.

I had despaired, however, of being able to adopt this last and best course, for I could not obtain from those in charge of the Asylum the exact census figures, although both Dr. Street, the Medical Superintendent, and Dr. Cheetham, the Assistant Medical Officer, did all in their power to help me. In particular the latter took the trouble to compile from the records of the Asylum, a complete enumeration of the inmate population on December 31st, 1891, classified according to ages and sexes, but the facts for 1881 were not attainable; I could only get the approximate total number. I had thought to make an approximate calculation as the best possible course. Finally, by the courtesy and kindness of Mr. Noel A. Humphreys, until recently Secretary of the Census Department, at the General Register Office, Somerset House, I was supplied with the exact figures of the *Inmate Population* of the Asylum, as enumerated at the two censuses of 1881, and 1891. To him I would wish to express my most grateful acknowledgments, as having thus enabled me to surmount this otherwise insuperable difficulty.

Therefore, although it may appear like laying violent hands on something so sacred as the fixed fact of the total population enumerated at the census, I have commenced by correcting the census enumerations of 1881 and 1891, by deducting from each, the respective numbers belonging to the inmate population of the Asylum.

To show still more clearly how the three different ways of dealing with the Asylum difficulty, must have led to very widely different results in the Life-Table, without going into details as to the separate age and sex groups, it may be stated generally with regard to the total *mean annual Death-rate* (calculated on the arithmetical mean population numbers).

$$\left.\begin{array}{r}\text{(1) Haydock Population}\\ \textit{including}\text{ Asylum Population}\end{array}\right\}\textit{ and }\left\{\begin{array}{l}\text{Haydock Deaths}\\ \textit{with}\text{ Asylum Deaths.}\end{array}\right.$$

will give a mean annual Death-rate of 20·35.

$$\left.\begin{array}{r}\text{(2) Haydock Population}\\ \textit{including}\text{ Asylum Population}\end{array}\right\}\textit{ and }\left\{\begin{array}{l}\text{Haydock Deaths}\\ \textit{without}\text{ Asylum Deaths}\end{array}\right.$$

will give a mean annual Death-rate of 16·95.

$$\left.\begin{array}{r}\text{(3) Haydock Population}\\ \textit{without}\text{ Asylum Population}\end{array}\right\}\textit{ and }\left\{\begin{array}{l}\text{Haydock Deaths}\\ \textit{without}\text{ Asylum Deaths}\end{array}\right.$$

will give a mean annual Death-rate of 17·43.

In the 1st case the result is 20 per cent too high, and in the 2nd case the result is 8·5 per cent too low.

The only other "Public Institution" in Haydock is the Cottage Hospital. The number of patients from other Districts, who may have been in the Hospital when the census of 1891 was taken (it was not in existence in 1881), could only have amounted to units, and may be considered a negligible quantity.

The same thing may be remarked with respect to the number of persons belonging to Haydock who may have been inmates of the Warrington Workhouse at the dates of the Censuses of 1881 and 1891.

The differences would have been so minutely fractional as to render it not at all worth while to have *taken* and *given* the labour involved in securing the exact facts. In any case these errors would have been in *opposite* directions and would have tended to counterbalance each other.

We must, then, take the numbers enumerated at the two Censuses of 1881 and 1891, corrected by excluding the inmate population of Haydock Lodge, as our *fixed points*. Of course, in Haydock, as in every other district, at the dates of the Censuses, there may have been persons belonging to Haydock absent at the time of the enumeration, while other persons properly belonging to other districts may have been temporarily staying in Haydock. These numbers will probably nearly counterbalance each other, but in any case it is impossible to get nearer to the truth.

The numbers in each age and sex group are given in detail in Table I.

Confining ourselves now to the *total* numbers, there are the following facts to go upon.

	Total Population.	Inmates of Haydock Lodge Asylum.	Remainder after deducting Asylum Inmates.
	CENSUS OF 1881.		
Males...	3,110 ...	... 110 ...	... 3,000
Females	2,753 ...	... 105 ...	... 2,648
Persons	5,863 ...	... 215 ...	... 5,648
	CENSUS OF 1891.		
Males...	3,509 ...	... 53 ...	... 3,456
Females	3,026 ...	... 73 ...	... 2,953
Persons	6,535 ...	... 126 ...	... 6,409

Thus we have a Population of 5,648 increasing in 10 years to 6,409.

The most obvious and simple method of arriving at the *mean* population for the period of 10 years is to simply add the two numbers together and divide by 2, thus getting the "Arithmetical mean."

$$\frac{5648 + 6409}{2} = 6028{\cdot}5 \text{ (or 6029)}$$

Unfortunately, however, this method is not so accurate as it is simple. By using it a double error is introduced.

(1) On the assumption that population increases by a constant "rate of increase," that is, in "geometrical progression," the *true* mean must necessarily be *less* than the arithmetical mean.

(2) The date of the Census of 1881 having been $\frac{1}{4}$ year after the *beginning*, and the date of the Census of 1891 having been $\frac{1}{4}$ year after the *ending* of the 10 calendar years 1881-90, this fact also would tend in an *increasing* population to make the mean of the Census interval, however calculated, *greater* than the mean of the 10 years 1881-90.

In order to find the *true* mean population for the 10 years 1881-90 the following formula has to be employed.

Let P = population at the Census of 1881, and r P = population at the Census of 1891, and r = "rate of increase" per unit for the 10 years, then

$$\frac{\text{r P} - \text{P}}{\text{r}^{\frac{1}{40}} \times \text{Hyperbolic Log. r}} = \text{true mean population.}$$

"r" is found to be = 1·134738

Therefore the formula works out thus:

$$\frac{6409 - 5648}{1{\cdot}003165 \times 0{\cdot}1264018} = \frac{761}{0{\cdot}1268018} = 6001{\cdot}5$$

Or, the same result is obtained from the arithmetical mean of the two Census populations by dividing it by the factor obtained from the following formula:

$$\frac{\text{r}^{\frac{1}{40}} \times (\text{r} + 1) \times \text{Hyperbolic Log. r}}{2\,(\text{r} - 1)} = \frac{1{\cdot}003165 \times 2{\cdot}134738 \times 0{\cdot}1264018}{2\,(1{\cdot}134738 - 1)} = \frac{{\cdot}27068874}{{\cdot}269476} = 1{\cdot}004500$$

There has been *no real need*, however, for working out either of these complicated formulæ, for in the last Decennial Supplement of the Registrar-General there is given a Table, called "Table P," by means of which the factor obtained by the last formula may be deduced for any rate of increase or decrease, by a simple calculation.

From this Table P the factor corresponding to r = 1·134739 is found to be = 1·004501

$$\text{and } \frac{6028{\cdot}5}{1{\cdot}004500} = 6001{\cdot}5 \text{ (or 6002)}$$

The results therefore are found to exactly correspond.

Having thus obtained the true mean total population number, it remains to get the true mean numbers for the different age-groups for each sex. For arriving at this result several methods suggest themselves.

(1) The numbers might have been *approximately* obtained by dividing *each* of the arithmetical means of each age-group, shown in Table I, by the same factor previously applied to the arithmetical mean of the total population, viz: 1·004500.

It is obvious that the sum of the numbers thus obtained would have been equal to 6001·5.

For to take a simple numerical example:—

$$\frac{2}{2} + \frac{4}{2} + \frac{8}{2} = \frac{2 + 4 + 8}{2} = 7$$

(2) Another way would have been to deduce, by means of the "Table P" already referred to, the *separate* factors of correction corresponding to the respective rates of increase or decrease in *each* of the age-groups of the population, and then to divide the respective arithmetical means by their respective factors. It may be open to some doubt, however, whether the same law of constant rate of increase, which is true when applied to the *whole* population, may be equally true when applied to some, at least, of the separate age-groups.

(3) The method which has been finally adopted is what may be called the "method of mean proportions." The steps in this are as follows:—

(*a*) The proportion *per million* for each age and sex group of the population is calculated for each of the two censuses.

(*b*) The assumption is made that, in the interval between the two censuses, the proportion in each age and sex group of the population has *changed uniformly*—that is supposing in some particular age-group the proportion is 10 per cent. at the *first* census, and 20 per cent. at the *second* census, then the proportion at the *middle* of the 10 years interval, *i.e.*, at the end of 5 years, would be 15 per cent. Now the middle of the 10 years, 1881-90, as has already been pointed out, does not exactly correspond to the middle of the interval between the two censuses, but the middle of the 10 calendar years, 1881-90, comes at 4$\frac{3}{4}$ years after the first census (that of 1881)—that is, we have to take $\frac{19}{40}$ of the *total* change in proportion from one census to the next. A convenient way of working out the calculations is this:—add together the proportions per million already found for each age-group for the two censuses, and divide by 2. This gives the mean proportion at the end of 5 years after the earlier census. Take the difference between this and the proportion at the earlier census, and this gives the change of proportion in 5 years. If the proportion has been an *increasing* one, $\frac{1}{20}$ of this difference must be *subtracted* from the mean proportion already found, and if it has been a *decreasing* one, $\frac{1}{20}$ of the difference must be *added* to the mean proportion.

(*c*) Having thus found the proportion per *million* for each age and sex group at 4$\frac{3}{4}$ years after the earlier census, it is easy to calculate what the proportions would be in the total population of 6,002.

The numbers thus calculated for each age and sex group are given in Table II.

The sum of the numbers in the male age-groups is 3,211·070, and of the numbers in the female age-groups 2,790·591, the total sum being 6,001·661. Therefore we have a total mean population of 6,002, consisting of 3,211 males and 2,791 females. It will be noted that the calculation has not come out with *absolute* exactitude, but its fractional differences are so minute as to be negligible.

In order to see how the respective methods suggested work out, separate calculations for *males* have been made by methods (1) and (2), and the results have been compared with each other. They are such as to shew that method (3) which has been adopted is preferable to the others. To economise space the Table of comparision has been left out.

II.—DEATHS.

Having now dealt with the population numbers, it remains to explain how the numbers in the Table of Deaths (see Table III.) have been arrived at.

The Monthly Returns of the District Registrar of Births and Deaths for the 10 years have been carefully gone over twice, and the Deaths classified into their respective groups according to ages and sexes. Before the final additions, each year's figures have been carefully checked.

There are, however, the following points to be alluded to:—

(*a*) The Deaths of the inmate Population of Haydock Lodge Asylum, as has been already explained, have been excluded. They are only set down in the Table to show still further how greatly they would have affected the results, had they been included. I had considerable trouble, however, in a discrepancy which I found to exist. In the Registrar-General's Decennial Report the number of Deaths among the inmate population of Haydock Lodge Asylum is given as 211 in the 10 years, 1881–90, whereas, according to the Monthly Returns in my possession, the number is 215. By the further help of Dr. Cheetham, of the Asylum, I was enabled to identify 4 deaths as having occurred among the resident staff, and to transfer them to their proper groups among the Haydock deaths.

(*b*) A certain number of persons died in Haydock during these 10 years who properly belonged to outside districts. These were chiefly Deaths of coal-miners meeting with immediately fatal injuries while at their work, or Deaths occurring at the Cottage Hospital.

These have, of course, to be excluded.

(*c*) On the other hand it is equally necessary to *include* Deaths of persons properly belonging to Haydock, but dying elsewhere. This has been done in so far as is possible. A Special Return has been obtained from the Superintendent Registrar for the district, of the deaths occurring in the Workhouse at Warrington of former Haydock residents. There remains, however, still some uncertainty as to what other deaths may have occurred in other districts of persons properly belonging to Haydock, and as has been already pointed out in the text of the preceding Annual Report, until something corresponding to a "Clearing-House" is established for supplying such information to all Medical Officers of Health, this uncertainty must always be a difficulty in the way of securing that rigid and minute accuracy which is so greatly to be desired.

In the meantime I can only remark that all has been done that could be done to secure exactitude.

The *foundations* of the Life-Table have now been laid in having determined—

(*a*) The true mean population numbers grouped according to age and sex.

(*b*) The corrected numbers of deaths arranged in corresponding groups.

The use which has to be made of these figures is to find out what is the chance or probability of surviving from one age to the next.

The series of fractions representing these probabilities is the SKELETON or SUPPORTING FRAMEWORK on which the whole Life-Table is to be built up. Much labour is involved, as has already appeared, in laying the foundations for these "probabilities," but when once they are calculated, the succeeding labour, which is also by no means inconsiderable, is simply a matter of routine calculation.

These probabilities are to be worked out:—

(*a*) Either *directly* from the Population numbers and the numbers of Deaths; or,

(*b*) *Indirectly* from the "rate of mortality" per unit or per thousand.

Let a case first be considered where we know (1) the number surviving at exactly a certain age at the beginning of the year; (2) the number dying during the year.

To take a simple numerical example. Suppose there are 10 infants born on the same day, and during the first year of life 2 die, there would be 8 chances out of 10 that any individual infant would live, and 2 chances out of 10 that the same infant would die. Thus the probability of surviving to the end of the first year of life would be represented by the fraction $\frac{10-2}{10} = \frac{8}{10} = \cdot 8$, which would be therefore the probability of surviving from age 0 to age 1; and if we had to calculate how many out of a million at birth would survive to the end of the first year of life, it would be done by multiplying a million by the fraction ·8, and 800,000 would be the number obtained, or, to put it generally, given "P" the population at exact age "x" at the beginning of the year, and "d" the number dying before the end of the year, then the chance of surviving from age x to age $x + 1$ is represented by the fraction $\frac{P-d}{P}$.

However, the population numbers, as enumerated at each Census, do not give us the numbers of persons *at the beginning* of the several years or age-periods, but the *numbers at all ages* between certain fixed points. Thus, if according to the Census enumeration, 10 children are returned as living at age 4–5—this means that they are of any age between the beginning and the end of the fifth year of life. Now if two of these children are returned in the death register as dying at age 4–5, the problem of calculating the chance of survival becomes more complicated. In order to solve it, we must assume two things.

(1)—That at the middle of the year the average age of these children is $4\frac{1}{2}$ years.

(2)—That the number of deaths is evenly distributed during the year, half occurring in the first half of the year, and half in the second half of the year.

Now on these assumptions which, *when large numbers are dealt with*, may be considered approximately true for any year in life *except the first*, the number of survivors at the beginning of the year would be 10 + 1, and at the end of the year 10—1, and the chance of surviving to the end of the year would be expressed by the fraction $\frac{10-1}{10+1} = \frac{9}{11} = \cdot 818181$.

And if we wanted to calculate how many out of a million at age 4 would survive to age 5, it would be done by multiplying the million by ·818181, the result being 818181.

To put the thing in a general form—If "P" be the number returned at the Census as living between any age x and the next age $x + 1$, they must be considered to be, at the middle of the year, of the average age of $x + \frac{1}{2}$; and if "d" be the number of deaths for the year at age x to $x + 1$, then the chance of surviving from age x to age $x + 1$ is expressed by the fraction $\frac{P-\frac{1}{2}d}{P+\frac{1}{2}d}$

In the above example we have worked *directly* from the population and deaths, but if we have given the rate of mortality per unit at age x to $x + 1$, which we may call "m," then the chance of surviving from age x to age $x + 1$, is expressed by the fraction $\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}$ or, if we have given the death-rate per thousand, which we may call "D," then the probability of surviving from age x to age $x + 1$ is expressed by the fraction $\frac{1000-\frac{1}{2}D}{1000+\frac{1}{2}D}$

It will be obvious that these two last fractions are really only different ways of expressing the fraction $\frac{P-\frac{1}{2}d}{P+\frac{1}{2}d}$ as the rate of mortality per unit or per thousand will have already been deduced from "P" and "d." Thus, $m = \frac{d}{P}$

After these very long preliminary explanations, it remains now to proceed to calculate from the data the probabilities of surviving one year for each of the separate years or age-periods of life. In all Life-Tables it is necessary to make these calculations *separately* for the first five years of life.

But here another difficulty arises. Although at the Census enumerations the numbers are given of those living for each of the years from 0–1 to 4–5, these numbers are found to be *altogether unreliable*. There is so much misstatement of age, that more are returned as surviving at some of the later ages than at preceding ages, which is impossible. The Census numbers, then, have to be discarded. We can only use them to determine the *total mean population for the age-period* 0–5. This may be taken as a *fixed* and fairly certain basis to work upon.

Having this number to work from, the numbers living at the separate years from 0–1 to 4–5 have to be determined by processes of calculations from the numbers of Births and Deaths under 4 years of age, returned by the District Registrar for the years 1876–80. These numbers have had to be partly obtained by a Special Return from the Superintendent Registrar. They are given in Table IV.

The following is an explanation of the method to be employed:—

The deaths under one year of age in the 10 years, 1881–90, must necessarily occur out of the whole number born in the 9 years, 1881–89, and out of *part* of those born in the year 1880 and *part* of those born in the year 1890. The deaths under one year in the 10 years, 1881–90, may therefore fairly be taken as occurring out of $\frac{1}{2}$ Births in 1880 + Births in 1881–89 + $\frac{1}{2}$ Births in 1890.

The numbers of births to be reckoned should, in strict accuracy, be exactly those occurring in the *last* half of the year 1880 + all those in the years 1881–89 + those occurring in the *first* half of 1890; but as the returns made by the District Registrar only give the *total* numbers of male and female births for each calendar year, it has to be assumed that these births are evenly distributed between the two halves of the year.

In the same way the deaths under 1 year in the 10 years, 1880–89, may be taken as occurring out of ½ Births in 1879 + Births in 1880–88 + ½ Births in 1889, and subtracting these deaths from the total number of births out of which they occurred, will give the number of children aged 1 year out of whom the deaths occurred in 1881–90 of children between 1 and 2 years of age.

Proceeding in a similar way for the other years, the following results are obtained.

CALCULATION FOR MALES.

(1) For mean annual number AT BIRTH in the 10 years, 1881–90.

Years ½ 1880 + 1881–89 + ½ 1890
Numbers 57 + 1101 + 67 = 1225, and $\frac{1225}{10} = 122{\cdot}5$

(2) For mean annual number AT 1 YEAR OF AGE in the 10 years, 1881–90.

FROM BIRTHS IN—

Years ½ 1879 + 1880–88 + ½ 1889
Numbers 67·5 + 1093 + 61 = 1221·5
deduct deaths under 1 year in 10 years, 1880–89 = 172
remainder = 1049·5, and $\frac{1049{\cdot}5}{10} = 104{\cdot}95$

(3) For mean annual number AT 2 YEARS OF AGE in the 10 years, 1881–90.

FROM BIRTHS IN—

Years ½ 1878 + 1879–87 + ½ 1888
Numbers 69 + 1107 + 60·5 = 1236·5
deduct deaths under 1 year in 1879–88 = 160
and deaths from 1 to 2 years in 1880–89 = 64
remainder = 1012·5, and $\frac{1012{\cdot}5}{10} = 101{\cdot}25$

(4) For mean annual number AT 3 YEARS OF AGE in the 10 years, 1881–90.

FROM BIRTHS IN—

Years ½ 1877 + 1878–86 + ½ 1887
Numbers 59·5 + 1115 + 75 = 1249·5
deduct deaths under 1 year in 1878–87 = 166
and deaths from 1 to 2 years in 1879–88 = 67
and deaths from 2 to 3 years in 1880–89 = 25
remainder = 991·5, and $\frac{991{\cdot}5}{10} = 99{\cdot}15$

(5) For mean annual number AT 4 YEARS OF AGE in the 10 years, 1881–90.

FROM BIRTHS IN—

Years ½ 1876 + 1877–85 + ½ 1886
Numbers 72·5 + 1114 + 60 = 1246·5
deduct deaths under 1 year in 1877–86 = 160
and deaths from 1 to 2 years in 1878–87 = 67
and deaths from 2 to 3 years in 1879–88 = 25
and deaths from 3 to 4 years in 1880–89 = 15
remainder = 979·5, and $\frac{979{\cdot}5}{10} = 97{\cdot}95$

A precisely similar calculation having been also made for females, the following results are obtained:—

	MALES.	FEMALES.
At Birth	122·5	119·2
,, Age 1	104·95	106·7
,, Age 2	101·25	103·35
,, Age 3	99·15	101·2
,, Age 4	97·95	100·5
Totals	525·80	530·95

It must be carefully noted that these total numbers give, NOT the population numbers at all ages, from Birth to 1, from 1 to 2, &c., but the numbers actually starting at Birth, at 1 year of age, at 2 years of age, &c.

Now the numbers arrived at from the Census enumerations (see Table II.) as the mean total population, *at all ages*, for the age-period 0–5, are

Males	491·992
Females	497·925

These numbers represent the totals given by calculation in the above table, *after half a year's mortality*, as well as *somewhat altered by migration*, that is, by children coming or going from the District.

In order to make the two sets of figures correspond—

(1) Either the calculated numbers in the table must be *brought on* to the middle of the years by subtracting half the mortality for the several years.

(2) Or the mean census population numbers must be *carried back* half a year by restoring the numbers who have died in the first half of the years of life.

The latter method may be considered the best to adopt, as being the most direct, and as enabling us to make the calculation of the chance of survival by the more simple formula $p = \frac{P-d}{P}$

At age 0—1, more than half the mortality occurs in the first half of the year of life; thus in Haydock during the 10 years 1881-90, out of a total of 160 deaths of *male* infants at age 0—1, 114 were under six months of age, and out of a total of 147 deaths of *female* infants at age 0—1, 96 were under six months of age. At each of the other years it may be fairly assumed that the mortality is evenly spread, that is, half in the first six months and half in the second six months of the year of age. So that for males (see Table III.) the half year's mortality among children at age-period 0—5 has been $11·4 + \frac{5·6+2·0+1·5+0·8}{2} = 16·35$, and $491·992 + 16·35 = 508·342$, and for females the half year's mortality among children at age-period 0—5 has been $9·6 + \frac{5·3+2·6+1·7+0·7}{2} = 14·75$, and $497·925 + 14·75 = 512·675$.

The differences which still remain between the numbers 525·80 and 508·342 for males and 530·95 and 512·675 for females, will represent the alterations made by migration.

To eliminate these differences the numbers 508·342 for males, and 512·675 for females, must be divided up proportionately to the respective numbers which collectively make up the totals of 525·80 and 530·95 in the table obtained by direct calculation from the numbers of births and deaths.

This having been done we arrive at the following results:—

AGE.	Number at each age out of whom the deaths occurred.		Number of deaths during 1881-90 in each year of age.	
	MALES.	FEMALES.	MALES.	FEMALES.
At Birth... ...	118·433	115·097	16·0	14·7
Age 1	101·465	103·027	5·6	5·3
Age 2	97·888	99·793	2·0	2·6
Age 3	95·858	97·717	1·5	1·7
Age 4	94·698	97·041	0·8	0·7
Totals ...	508·342	512·675	25·9	25·0

From this table, therefore, it is obvious that the facts are provided for DIRECTLY calculating the probabilities (or chances) of survival for each of the first five years of life by the formula $\frac{P-d}{P}$ that is, $\frac{\text{Population}-\text{Deaths}}{\text{Population}}$ = Probability of surviving one year.

The respective chances of living one year are as follows:—

	MALES.	FEMALES.
At Birth	$\frac{118{\cdot}433-16}{118{\cdot}433}={\cdot}86491$	$\frac{115{\cdot}097-14{\cdot}7}{115{\cdot}097}={\cdot}87223$
At Age 1	$\frac{101{\cdot}465-5{\cdot}6}{101{\cdot}465}={\cdot}94481$	$\frac{103{\cdot}027-5{\cdot}3}{103{\cdot}027}={\cdot}94855$
At Age 2	$\frac{97{\cdot}888-2}{97{\cdot}888}={\cdot}97957$	$\frac{99{\cdot}793-2{\cdot}6}{99{\cdot}793}={\cdot}97395$
At Age 3	$\frac{95{\cdot}858-1{\cdot}5}{95{\cdot}858}={\cdot}98435$	$\frac{97{\cdot}717-1{\cdot}7}{97{\cdot}717}={\cdot}98261$
At Age 4	$\frac{94{\cdot}698\quad 0{\cdot}8}{94{\cdot}698}={\cdot}99155$	$\frac{97{\cdot}041-0{\cdot}7}{97{\cdot}041}={\cdot}99279$

I cannot pretend to have arrived at full comprehension of these last somewhat complicated processes of calculation by my own unaided efforts. I had to some extent understood them by reading the introduction to Dr. Tatham's Manchester Life-Tables, but, for having finally obtained a clear mental vision of the *reasons* and methods of the calculations, I am indebted to help personally given to me by Mr. A. C. WATERS, of the General Register Office, who has done the work of calculating the recently issued Life-Tables for England and Wales, and to whom I cannot be sufficiently grateful for having taken considerable trouble in giving me lucid explanations of my difficulties. Not only on this point, but on others to be hereafter alluded to he has given me help most invaluable.

For the remaining age-periods the probabilities of survival have to be worked out, at least, in the first instance, *in groups of years*, from the true mean numbers of the living *at all ages*, as deduced from the Census enumerations, and the numbers of deaths returned as having occurred at the same age-periods.

When these are determined, it remains to be considered whether to make a *complete* Life-Table, that is, one for every separate year of Life, or whether to proceed by what is called the "short" method, that is, to construct a Life-Table in which the mean chances of of surviving one year during each of the respective age-periods, are taken as being true for *each* year of the respective age-periods.

The labour involved in making a complete Life-Table is very great indeed, as the separate probabilities of survival for the respective years of each age-period have to be, as it is termed, "interpolated" in one or other of two ways.

(*a*) By the "method of finite differences" applied to the logarithms of the numbers of Population, and of Deaths, as existing at the fixed points, at the beginning and end of each age-period. This is also called the "analytical" method, and involves calculations of exceeding difficulty and labour.

(*b*) By the "graphic" method in which the numbers for the separate years are determined by a process of geometrical construction, which consists in drawing "curves" through a series of parallelograms which have been drawn to scale. This also, while to those skilled in its use it may be an easier and shorter method than the former one, is one not to be contemplated lightly by a beginner.

Had there been any adequate advantage to be gained by it, I was prepared to have undertaken the work by one or other of these methods.

However, I had the advice given by Mr. A. C. Waters that for so small a population the labour of doing this would have been thrown away.

When the "short" method is adopted, first devised by Dr. Farr, 10-yearly periods are taken at least after the age of 25. A method a little more elaborate is to take 5-yearly periods.

While the short method by 10-yearly periods gives approximately true results, as to the correct "expectation of life" at the earlier ages, later on, and especially after the age of 60, it gives results considerably too high.

To take a simple numerical example in illustration of this point.

Suppose that there are, at the age of 65, survivors numbering 16, and that the probability of surviving 1 year for each of the years to age 75 has been calculated as ·87055.

Then the number of survivors at age 75 would have to be calculated by multiplying 16 by the fraction ·87055 ten times, that is by $(\cdot 87055)^{10}$.

By the aid of logarithms the value of the 10th power of ·87055 is readily found to be 0·25 or $\frac{1}{4}$. The number of survivors at age 75 would then be $16 \times \frac{1}{4} = 4$.

In reckoning up the total number of years lived through during the 10 years, we have first of all 4 who have survived to the end, and they will have lived collectively $4 \times 10 = 40$ years. We must suppose then that the 12 who have died during the interval have on the average lived through half the interval, so that they will have collectively lived $12 \times 5 = 60$ years. Adding these to the 40 we get $40 + 60 = 100$ years, as the total number of years lived through by a mean population of $\frac{16+4}{2} = 10$ persons during the ten years.

Now, if the probabilities of survival have been separately calculated for the 5-yearly intervals 65-70 and 70-75, the former will be found to be somewhat greater, and the latter somewhat less than ·87055, but the *mean* probabilities for the two periods will perhaps not greatly differ from ·87055.

To find the number of survivors at age 70, the number 16 must be multiplied five times by the fraction of probability, that is, by $(\cdot 87055)^5$. The value of this expression is found to be 0·5 or $\frac{1}{2}$. The number of survivors at age 70 will be therefore $16 \times \frac{1}{2} = 8$. And in the same way the number of survivors at age 75 will be $8 \times \frac{1}{2} = 4$. In calculating the total years of life lived through during the age-period 65-70, we must of course allow 5 years for each of the survivors at the end of the period, and $2\frac{1}{2}$ years for those who die *during* the period, or what comes to exactly the same thing, allow 5 years each to the *mean* number living during the period.

The result will work out thus:—

$$\left.\begin{array}{l}\text{Years of life lived through}\\ \quad\text{during the age-period}\\ \quad 65—70.\end{array}\right\} = \frac{16 + 8}{2} = 12, \text{ and } 12 \times 5 = 60.$$

$$\left.\begin{array}{l}\text{Years of life lived through}\\ \quad\text{during the age-period}\\ \quad 70—75.\end{array}\right\} = \frac{8 + 4}{2} = 6, \text{ and } 6 \times 5 = 30.$$

The sum of 60 + 30 = 90, a less result by 10 than by calculating for the 10 years by *one* stage.

It is therefore much better to work out the Life-Table for 5-yearly periods, as a nearer approximation to the perfect results of a complete Life-Table is thus obtained.

Here again, however, another of the difficulties so constantly arising in this work is met with.

There is a very great tendency, especially at the later periods of life, for ages to be stated "in round numbers," both at the Census enumerations, and in the Death registers; so that ages are stated to be 50, 60, or 70 as the case may be, when in reality they are above or below these even numbers.

Therefore if the probabilities of survival are *directly* calculated from the numbers of Population and of Death returns for these *even* ages, the results are unreliable. In both of two previous complete Life-Tables for 5-yearly periods which I had worked out, and had in succession to discard as imperfect, I had calculated the probabilities at the even ages from the Census population and Death returns without correction, and the results were "rugged."

In this third attempt I have employed a method for the knowledge of which I am again indebted to Mr. A. C. Waters, who not only told me of the method but fully explained how to use it.

This method may therefore be considered to be as perfect a compromise as can be made between the crudeness of the "short" Life-Table for 10-yearly periods, and the perfection of a complete Life-Table, which involves such enormous labour in its construction.

For the sake of brevity, explanation will only be given of the method as applied to the *Male* section of the Life-Table, it being understood that the data provided for the *Female* section have been dealt with in exactly the same way.

It should be clearly understood at the outset that although the Population and Death figures for the even ages are to be obtained by an elaborate calculation, the TOTAL numbers between the fixed points 5-15, 15-25, 25-35, etc., ARE NOT AT ALL ALTERED. They are simply each distributed into two portions, 5-10 and 10-15, etc., in such proportions as certainly give a much nearer approximation to the truth than is given by the crude recorded numbers.

The calculation may be briefly described as a simple application of the same "method of finite differences" which, in a much more elaborate and complicated form, is used in "interpolating" the numbers for the separate years in the complete Life-Table.

The method may be applied either to the Population and Deaths *separately*, or more conveniently to the Population and Deaths *combined*, as then the resulting numbers will *at once* enable the probabilities of survival to be calculated by the formula already described,

$$\frac{P - \frac{1}{2}d}{P + \frac{1}{2}d}$$

First of all from Tables II. and III. the numbers of the living and of the dying at the 10-yearly age periods are to be set down as follows:—

MALES.

Ages.	Mean Population.	Mean Annual Deaths.
5—15	805·820	3·4
15—25	646·598	1·7
25—35	499·645	2·7
35—45	336·092	4·1
45—55	217·158	3·8
55—65	137·451	4·4
65—75	62·720	5·5
75—85	13·036	2·6
85—95	0·558	0·5

Then by simple addition and subtraction the above figures are combined as follows:—

Ages.	Population + ½ Deaths.	Population — ½ Deaths.
5—15	807·520	804·120
15—25	647·448	645·748
25—35	500·995	498·295
35—45	338·142	334·042
45—55	219·058	215·258
55—65	139·651	135·251
65—75	65·470	59·970
75—85	14·336	11·736
85—95	0·808	0·308

The next step is to construct from the above numbers two other columns by *successive additions, beginning from below.*

	Symbol for each number.	Population + ½ Deaths.	Population — ½ Deaths.
At age 5 and upwards	u_5	2,733·428	2,704·728
,, 15 ,, ,,	u_{15}	1,925·908	1,900·608
,, 25 ,, ,,	u_{25}	1,278·460	1,254·860
,, 35 ,, ,,	u_{35}	777·465	756·565
,, 45 ,, ,,	u_{45}	439·323	422·523
,, 55 ,, ,,	u_{55}	220·265	207·265
,, 65 ,, ,,	u_{65}	80·614	72·014
,, 75 ,, ,,	u_{75}	15·144	12·044
,, 85 ,, ,,	u_{85}	0·808	0·308

It will now be easily apparent that if by any process of calculation we can "interpolate," in the spaces between the above numbers, the means corresponding to u_{10}, u_{20}, u_{30}, and so on to the bottom of each column; then by successive *subtractions beginning from the top*, we shall obtain the intercepted numbers corresponding to age 5—10, age 10—15, age 15—20, age 20—25, &c.

Now there are various ways in which such "means" might be calculated, from the most simple to the most complex.

(1) u_5 and u_{15} might be simply added together and the sum divided by 2. This would give the "arithmetical mean."

(2) The Logarithms corresponding to u_5 and u_{15} might be added together, and their sum divided by 2. The corresponding number to the resulting Logarithm would be the "geometric mean." This is the "method of mean squares."

If we were dealing only with a series of two numbers, the intermediate term would have to be found by one or other of these two methods.

But if we have a series of four terms, a more accurate mean to the two middle terms of the series can be obtained by a calculation based on all four terms of the series.

Thus, having given the four terms, u_5, u_{15}, u_{25}, u_{35}, the "method of finite differences" gives the following formula for finding u_{20} :—

$$u_{20} = \frac{9}{16} (u_{15} + u_{25}) - \frac{1}{16} (u_5 + u_{35})$$

or, this formula may be expressed in a simpler form for working with, as follows:—

$$u_{20} = \frac{10 (u_{15} + u_{25}) - (u_5 + u_{15} + u_{25} + u_{35})}{16}$$

That is to say :—Take ten times the sum of the two *middle* terms, and subtract from this the sum of *all four terms*; then one-sixteenth of the remainder is the CENTRE term, u_{20}, required.

(3) This formula applied to the numbers gives a modified *arithmetical* mean.

(4) The same formula applied to the *Logarithms* of the numbers, gives a modified *geometrical* mean, and is THE METHOD TO BE PREFERRED AS BEING THE MOST EXACT.

The next step therefore is to translate the numbers of the preceding Table into their corresponding Logarithms, and to put them in a column LEAVING A BLANK SPACE BETWEEN EACH PAIR, and then to interpolate Logarithms in the blank spaces by the following formula:—

$$\text{Log. } u_{20} = \frac{10 (\text{Log. } u_{15} + \text{Log. } u_{25}) - (\text{Log. } u_5 + \text{Log. } u_{15} + \text{Log. } u_{25} + \text{Log. } u_{35})}{16}$$

and so on to Log. u_{70}, &c., the numbers indicating the terms of the series being successively altered by the addition of 10 to each.

It will be obvious, however, that this formula cannot be applied to obtain Log. u_{10} and Log. u_{80}, as they are too near respectively to the beginning and ending of the series. They may be obtained, however, by the following formulæ (leaving out the prefix of Log. to each term).

$$u_{10} = \frac{u_5 + u_{25}}{4} + 1\tfrac{1}{2}\, u_{15} - u_{20}$$

$$u_{80} = \frac{u_{85} + u_{65}}{4} + 1\tfrac{1}{2}\, u_{75} - u_{70}$$

It will be apparent, therefore, that these formulæ cannot be applied until u_{20} and u_{70} have been already calculated by the preceding formula.

All the above formulæ are original ones devised by Mr. A. C. WATERS.

It will serve no useful purpose to put down the actual figures of the Logarithms. It must be taken for granted that they have been worked out, and finally translated back into their corresponding numerical values. This having been done the following series of numbers are obtained. It must be borne in mind that the symbol u_5 means AT AGE 5 AND UPWARDS, and so on.

POPULATION + ½ DEATHS				POPULATION — ½ DEATHS.			
Symbol.	Numbers corresponding to Logarithms.	Age-periods.	Intercepted numbers for each age-period obtained from preceding column by *successive subtractions beginning from the top.*	Symbol.	Numbers corresponding to Logarithms.	Age periods.	Intercepted numbers obtained from preceding column by successive subtractions beginning from the top.
u_5	2733·428	5—10	425·912	u_5	2704·728	5—10	423·770
u_{10}	2307·516	10—15	381·608	u_{10}	2280·958	10—15	380·350
u_{15}	1925·908	15—20	342·265	u_{15}	1900·608	15—20	341·410
u_{20}	1583·643	20—25	305·183	u_{20}	1559·198	20—25	304·338
u_{25}	1278·460	25—30	271·400	u_{25}	1254·860	25—30	270·253
u_{30}	1007·060	30—35	229·595	u_{30}	984·607	30—35	228·042
u_{35}	777·465	35—40	185·941	u_{35}	756·565	35—40	183·839
u_{40}	591·524	40—45	152·201	u_{40}	572·726	40—45	150·203
u_{45}	439·323	45—50	119·688	u_{45}	422·523	45—50	117·684
u_{50}	319·635	50—55	99·370	u_{50}	304·839	50—55	97·574
u_{55}	220·265	55—60	78·580	u_{55}	207·265	55—60	76·594
u_{60}	141·685	60—65	61·071	u_{60}	130·671	60—65	58·657
u_{65}	80·614	65—70	41·791	u_{65}	72·014	65—70	38·956
u_{70}	38·823	70—75	23·679	u_{70}	33·058	70—75	21·014
u_{75}	15·144	75—80	10·896	u_{75}	12·044	75—80	9·427
u_{80}	4·248	80—85	3·440	u_{80}	2·617	80—85	2·309
u_{85}	0·808	85—90	0·808	u_{85}	0·308	85—90	0·308

From this form of calculation it will easily be seen that if 2733·428 be the *total* number of population + ½ deaths FROM AGE 5 UPWARDS, and if 2307·516 be the *total* corresponding number FROM AGE 10 UPWARDS, then 2733·428 — 2307·516 = 425·912 will be the number required for the Age-period 5—10, and so on.

It will also be evident, that as 425·912 + 381·608 = 807·520, which is the total number for Age-period 5—15, with which we started as a *fixed point*, that there has been no interference with the foundation numbers—BUT ONLY A MORE EVEN DISTRIBUTION.

It will also be apparent that after these devious wanderings through mazes of figures, we have at last the numbers required for calculating the probabilities of surviving one year for each of the 5-yearly periods.

Thus by the formula $\frac{P - \frac{1}{2}d}{P + \frac{1}{2}d}$ the chance of living one year during the age-period, 5—10, is found by the fraction $\frac{423·770}{425·912} = ·99497$. Similarly the chance of survival for one year during the age-period, 10—15, is $\frac{380·350}{381·608} = ·99672$.

The remaining chances of living one year (for males) have been similarly calculated. Corresponding calculations having also been made for females, all the chances of survival, which, as has been already explained, are the *foundations on which the whole Life-Table is to be built up*, are to be found set down in parallel columns in the third section of the Life-Table "for Males and Females."

A brief explanation may, therefore, now be given of the way in which the Life-Table proper is to be constructed. It may be said, too, that all these calculations, which appear so formidable on first view, are to be made with ease, when once a facility is acquired in the use of Tables of Logarithms.

It is usual to construct the Life-Table in three sections:—

I. for Males. II. for Females. III. for Males and Females.

To start with Section I., for Males.

A blank sheet of paper is taken and ruled into 7 columns, with the following headings:—

1—Age, "x." Below this heading the numbers 0, 1, 2, 3, 4, 5, 10, &c., are placed.

It must be carefully noted that, as applied to the figures in the remaining columns, these numbers may have to be read "at Birth," "at Age 1," "at Age 2," &c., in the case of column 3, or "the year from Birth to Age 1," "the year from Age 1 to Age 2," &c.—that is 0—1, 1—2, &c.—and then afterwards 5—10, 10—15, &c., in the case of columns 2, 4, 5, and 6.

2—Number dying in each year or Age-period of 5 Years. The symbol for this is "d^x." This column would more logically come *after* the next column, as the numbers of the dying cannot be calculated until we know the numbers of the living. $d^x = l^x - l^{(x+1)}$ that is the number dying during age 0—1, is equal to the number at birth minus the number surviving at age 1. But it is customary for the sake of convenience to place it in this order.

3—Number supposed to have been originally born, and number surviving at each age (that is, at the beginning of each Year or Age-period of 5 Years). The symbol for this column is "l^x."

4—Mean population living in each year or Age-period of 5 Years. The symbol for this column is "P^x." In the case of intervals of one year $P^x = \dfrac{l^x + l^{(x+1)}}{2}$

When the intervals are 5-yearly, $P^x = \dfrac{l^x + l^{(x+5)}}{2}$

That is, the mean number living during the year or age-period is an arithmetical mean between the number at the beginning and the number at the end of the year or age-period.

5—Years of life lived in each year or Age-period of 5 Years.

In the case of 1-year intervals the numbers in this column are identical with those of column 4 (except in the case of the year 0—1), so that in the complete Life-Tables, one column does for both "Population" and "Years of life lived." But when 5-yearly periods are taken the number in column 4 has to be multiplied by 5, to get the number for column 5.

(I have the authority of Mr. A. C. Waters for saying that the difficult problem of arriving at the true number of years of life lived in 5-yearly periods is more correctly solved by taking the simple arithmetical mean than by some more elaborate ways. It would occupy too much space to go into a detailed mathematical discussion on this point.) (As an afterthought, some remarks relating to this question have been inserted at the end.)

6—Sum of the years of life lived IN each year or Age-period AND AFTERWARDS TO THE END OF THE TABLE.

This column cannot be constructed until the preceding column is completed. It may then be constructed (1) either by adding up all the numbers in column 5 and placing the sum opposite age 0, and then, beginning at the top, successively subtracting the individual numbers in column 5, or, (2) by successive additions of column 5 beginning from below. The result is in either case the same.

The symbol for this column is "Q^x."

7—“Mean after lifetime” or mean expectation of Life.

Symbol E^x, and $E^x = \frac{Q^x}{l^x}$

That is, if we know the whole number of years lived through *in* a certain age and *afterwards to the end of life* by a certain number of individuals, the division of this number by the number who collectively lived the years will give the *mean after lifetime.*

Being now provided with the sheet of paper ruled into columns with the headings just indicated, it only remains, before proceeding with the work, to determine with what number of Male infants we shall start, or suppose to be born on the same day. It really does not matter in so far as the ultimate results, to be obtained in column 7, are concerned, what number is taken. But for the sake of being able to easily construct the *third* section of the Life-Table, it is convenient to divide up a Million infants in the proportions of the total Male and Female Births which have happened in the District during the 10 years.

For Haydock these numbers are—

Births in the 10 Years 1881—90.

Males	1,235	Corresponding to—	509,488
Females	1,189		490,512
Total	2,424		1,000,000

Therefore, first of all the number 509,488 is set down in 3rd (or l^x) column of the blank *Male* Life-Table.

According to the form of the Table, just as in a building the foundations are out of sight, or as in the human body the supporting skeleton is not visible, so the “probabilities of survival,” to avoid needless repetition, are only set down in the third section of the Life-Table. We are already provided with the Logarithms corresponding to these probabilities (the p^x numbers) for they have been first of all obtained in the form of Logarithms, before they have been translated into common numbers.

So the first calculation is—

	Numbers.		Logarithms.
	509,488	=	5·7071255
×	·86491	= +	$\bar{1}$·9369684
	440,650	=	5·6440939

It is most convenient to proceed with this series of calculations right on to the bottom of the third (or l^x) column, adding in succession the several Logarithms and translating the results into their corresponding numbers, and to finally carefully check the numbers, for it is very disastrous, as I know to my cost, to afterwards find errors, which make it necessary to do a good part of the work over again.

When the 5-yearly periods are reached, *5 times the Logarithm of the chance of surviving one year must be used.* It must be carefully noted, between which two separate years, the original number is reduced to exactly one half.

The last number 39 in this column, in order to make the Life-Table correspond with the observed facts of the Census enumerations, has not been derived from calculation from its preceding number 4,842 but by a proportionate calculation. If so many survive at age 90 out of a mean total male population of 3,211, how many would survive out of 509,488 And then as according to the census returns no males were found to be living at age 95, these 39 have all had to die during the age-period 90—95.

Having now calculated and verified all the numbers in this column, the construction of columns 2, 4, and 5, may be proceeded with.

The number dying during age 0—1 is found by subtracting 440,650 from the original number 509,488 and = 68,838, so this number is set down at age 0—1 in the second column, and so on.

It will be apparent that the total of the numbers in column 2, should be exactly 509,488.

Then for column 3, $\frac{509,488 + 440,650}{2} = 475,069$ is the first number to set down in column 4. For this age 0—1, we cannot allow so much as an average of half-a-year for the 68,838 who have died during this year, as a much larger number of Infants die during this first year *under* 6 months of age than *over* 6 months.

The calculation has to be made from the OBSERVED FACT that the MEAN AGE AT DEATH, of the 160 Male Infants who died in Haydock during the 10 years 1881–90, was 3·8275 months. So that for the 68,838 Male Infants who die in the Life-Table during age 0—1, we must allow 3·8275 mns. × 68,838 = 21,956 years, instead of the 34,419 years to which they would have been entitled at the rate of half-a-year each. Therefore 440,650 + 21,956 = 462,606 is the number to be set down first in COLUMN 5. For the remaining single years of life the numbers in COLUMN 5 will exactly correspond with those in COLUMN 4 and when the 5-yearly periods are reached the numbers in COLUMN 5 will be those in COLUMN 4, multiplied by 5.

As has been already explained this comes to the same thing as allowing an average of half-a-year of life to those dying in the single years, and of 2½ years of life to those dying in the 5-yearly periods.

Full explanations have already been given of the mode of calculating the figures in COLUMNS 6 and 7, so that at last this part of this Life-Table is finally completed.

The *Female* section of the Life-Table is constructed in exactly the same way. As the 147 Female Infants who died in Haydock during the 10 years 1881–90 were found to have a *mean age at death* of 4·195 months, the first number in column 5 is obtained by adding 4·195 months × 62,648 = 21,901 years to 427,864 (the number of survivors at age 1) the sum being 449,765 years.

The last number 36 in the l^x column is obtained by a proportionate calculation similar to that described for the Male Life-Table. And, although from the Census enumeration it is *possible* that some of them may have lived to be over 100 years of age, they have been summarily disposed of before reaching that age; as from the merely fractional mean numbers deduced from the Census enumerations no reliable probabilities of survival could be calculated.

As regards the third section of the Life-Table for MALES AND FEMALES. In order to make comparison between the numbers of survivors of each sex, OUT OF AN EQUAL NUMBER at Birth, separate calculations have been made for Males and Females starting with a million of each sex at Birth.

These calculations have the further advantage of checking the similar calculations in column 3 of the MALE and FEMALE sections of the Life-Table. So that it is desirable to have gone through them *before* having proceeded further with the remaining columns of the separate Male and Female Tables.

The third l^x column is simply obtained by adding together the numbers of survivors at the respective ages, in the Male and Female separate Tables.

In the same way the combined Q^x column is obtained, so that if desired the formula $E^x = \frac{Q^x}{l^x}$ can be applied so as to get the mean "expectation of life" for *Persons.*

It is obvious by mere inspection that at Birth the mean "expectation of life" for *Persons* is 46·55 years.

In this connection it may be noted that it is remarkable how close an approximation to the above number is to be obtained by the application of a very simple formula devised by the late Dr. FARR, for calculating the mean "expectation of life" at Birth from the Birth-rate and Death-rate WHEN THESE RATES ARE CALCULATED ON THE TRUE MEAN POPULATION.

Calling the Birth-rate B, and the Death-rate D, according to the formula, "expectation of life" at Birth $= \frac{2}{3} \times \frac{1000}{D} + \frac{1}{3} \times \frac{1000}{B}$

Now calculated on the true mean population of 6,002, the mean annual Birth-rate for Haydock during the decennium 1881–90 was 40·39 per 1,000, and the Death-rate 17·38.

$$\text{and } \frac{2}{3} \times \frac{1{,}000}{17{\cdot}38} + \frac{1}{3} \times \frac{1{,}000}{40{\cdot}39} = 46{\cdot}61 \text{ years.}$$

a difference of only 0·06 from the number calculated from the Life-Table.

The Life-Tables are now completed. The "two-foot rule" is provided, and it is possible now to use it in making measurements and comparisons.

The practical uses of a Life-Table when finished are to make comparisons in three different ways with the corresponding numbers in other Life-Tables.

I. The probabilities of survival, or chances of living one year at the respective ages, are to be compared. This is the best test of the vitality of a community *at any given age*, for it is independent of what has gone before or what follows after, and depends on the rate at which people are dying *at that special period*. It has already been shown that the probability of survival can be calculated from the rate of mortality per unit by the formula $p = \frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}$; and *vice versâ*, the rate of mortality can be readily calculated from the probability of survival by the formula $m = \frac{2(1-p)}{p+1}$, in which "p" is the *known* quantity.

II. The numbers of survivors at any given age may be brought into comparison.

These numbers, however, depend on what has happened before, that is, on the rates of mortality *at preceding ages*.

III. The numbers expressing in years the mean after-lifetime, or "expectation of life" may be set side by side and the differences taken.

This test does not bring into account anything connected with the *preceding* mortality, but depends entirely on the rates at which people die *after* the age at which comparision is made.

The following are the Life-Tables which are available for comparision, and which *have all been calculated from the mortality in the same* 10 *years* 1881–90.

(1) The Life-Table for England and Wales contained in Part I. of the last "Decennial Supplement" of the Registrar General. This is based on the statistics for *the whole country*.

(2) The Life-Table for selected Healthy Districts of England and Wales contained in Part II. of the same Supplement. This is based on the mortality of 263 Registration Districts, with a mean aggregative population amounting to about one-sixth of the total population of England and Wales. The test for inclusion among these districts was, that after corrections had been made by the method which is to be shown afterwards as worked out for Haydock in table VI., their *mean total death-rates for persons* fell below 15 per 1,000 in the 10 years 1881–90.

As judged by this test Haydock would very far fall short of the standard set up, for as will be seen (see Table VI.) the *comparative* mean annual-death rate for this period of 10 years was *18·84* for Persons.

Indeed, there were only two districts in the whole of Lancashire found worthy of inclusion, viz., Garstang and Lunesdale.

(3) Besides the above there are Life-Tables for the following cities or towns:—

London, Manchester, Glasgow, Brighton, Liverpool,

which have been worked out by their respective Medical Officers of Health.

Perhaps the time may come when every Sanitary District will have its own Life-Table. Possibly some such may have been constructed for Districts as small as Haydock, but in the meantime, not having any knowledge of their existence, comparison must be restricted to such Life-Tables as are available.

The most important comparisons to be made will be first of all, on the one hand, with the Life-Table for the whole of England and Wales, and, on the other, with the selected Healthy Districts Life-Table; and then with Brighton as an important health-resort, and also with the *worst* of the three Life-Tables for Manchester which was published by Dr. TATHAM during his tenure of Office as Medical Officer of Health for that City.

First of all a Table may be given comparing the *chances of living one year* at various ages, for Males and Females, as found in the Haydock Life-Table, with the corresponding figures: *(a)* for England and Wales, and *(b)* for the selected Healthy Districts.

(It is to be understood, of course, that the nearer the fraction approaches to unity the *less* has been the rate of mortality at the respective ages.)

This may be appropriately followed by a Table comparing the respective *death-rates per thousand* for Haydock, at certain age-periods, with the corresponding figures for England and Wales, and for the Healthy Districts, although in neither of the Life-Tables have these death-rates been used *directly* in the calculations, but they have been, however, calculated from the *same data* as the Life-Table probabilities of survival. The comparative mortality will be thus made more immediately obvious.

Chances of living one year.

	MALES.			FEMALES.		
Ages.	England and Wales.	Haydock.	Healthy Districts.	England and Wales.	Haydock.	Healthy Districts.
0	·83896	·86491	·88085	·86887	·87228	·90650
5	99168	·99497	·99444	·99214	·99635	·99484
10	·99805	·99672	·99793	·99833	·99655	·99763
15	·99713	·99750	·99724	·99705	·99590	·99670
25	·99364	·99562	·99459	·99379	·99347	·99438
35	·98981	·98870	·99262	·99076	·99338	·99270
45	·98437	·98326	·98953	·98765	·98435	·99110
55	·97398	·97473	·98239	·97910	·97706	·98499
65	·94943	·93431	·96293	·95801	·95033	·96725
75	·89542	·86518	·91122	·90721	·84130	·91980

Table showing comparison of the mean annual death-rates per thousand for each sex in age-groups for Haydock, during the 10 years, 1881-90, with the corresponding rates—(1) For the whole of England and Wales; and (2) For the Selected Healthy Districts. The calculations having been made in each case on the TRUE MEAN POPULATIONS for the Decennium.

Ages.	MALES.			FEMALES.		
	* England and Wales.	Haydock.	Healthy Districts.	* England and Wales.	Haydock.	Healthy Districts.
0— 5	61·64	52·64	39·70	51·96	50·21	32·38
5—10	5·35	5·65	3·88	5·26	5·02	3·86
10—15	2·95	2·48	2·28	3·11	1·74	2·71
15—20	4·32	2·60	3·36	4·43	4·73	4·21
20—25	5·74	2·67	5·18	5·54	5·55	5·32
25—35	7·77	5·40	6·15	7·38	7·54	6·21
35—45	12·40	12·20	8·52	10·59	7·72	7·96
45—55	19·38	17·50	12·80	15·12	16·39	10·70
55—65	34·73	32·01	24·16	28·48	27·84	21·07
65—75	70·50	87·69	55·54	60·46	71·75	49·75
75—85	146·75	199·47	132·93	130·76	186·43	118·67
85 upwards	305·98	896·06	305·30	271·10	264·32	266·94

*—The death-rates for England and Wales here given are not those which were stated in Part I. of the Registrar-General's Decennial Supplement, but the amended rates in Part II. The former were calculated on the arithmetical means of the two census populations, the latter on the *true* mean population numbers.

The leading points to note with regard to the above Table in comparing Haydock with the whole of England and Wales are:—

1.—The much lower death-rate of Males at age 0—5.

2.—The remarkably lower death-rates of males at ages from 10 to 35.

3.—As regards Females the death-rate at age 0—5 is but very little lower. This fact, associated with the *excess* of Female births over Male births, will be found to explain succeeding anomalies as regards *numbers of survivors* at various age-periods and the *mean expectation of life* throughout.

4.—As regards both sexes the death-rates after age 65 become *considerably higher.*

It would occupy too much space to go into complete comparisons with all the other Life-Tables which have been mentioned, but in passing it may be as well to compare the death-rates at age 0—5, so as to show how largely the Life-Table results are affected by the mortality at this age-period.

Mean Annual Rates of Mortality at age-period 0—5 during the Decennium, 1881-90.

Age.	MALES.					FEMALES.				
	Haydock.	Brighton.	London.	Glasgow.	Manchester Township.	Haydock.	Brighton.	London.	Glasgow.	Manchester Township.
0—5	52·64	64·01	73·09	86·24	103·67	50·21	52·57	63·26	75·52	88·34
	Corresponding probabilities of survival at age 0 and at age 4.									
0	·86491	84608	·83608	·82531	·76926	·87228	·87672	·86172	·85318	·80841
4	·99155	·98863	·98557	·98001	·98222	·99279	·98857	·98600	·97995	·98063

The next particular in which comparison may be made is as to the *number of survivors* at various ages. The following Table shows such comparisons between *Haydock* and *England and Wales* and also the "*Healthy Districts.*"

Numbers of survivors out of an equal number, (a million), of each sex born at various ages.

Ages.	MALES.			FEMALES.		
	England and Wales.	Haydock.	Healthy Districts.	England and Wales.	Haydock.	Healthy Districts.
1	838,964	864,905	880,855	868,874	872,280	906,497
5	751,494	781,289	826,862	783,244	786,120	855,168
10	733,477	761,840	811,820	766,151	771,867	840,057
15	726.194	749,435	803,060	759,062	758,661	829,273
25	693,809	729,931	771,435	724,788	720,500	793.029
35	639,645	690,244	723,851	670,992	667,534	743,125
45	564,437	608,555	663,894	604,007	617.295	686,542
55	462,981	510,539	581,586	516,375	523.762	614,089
65	322,482	367,170	452,117	385,503	394,601	494,065
75	153,890	143,897	253,622	204,208	177,431	294,100

The following are the ages at which the original million will have been reduced to one half:—

	England and Wales.	Haydock.	Healthy Districts.
Males - -	51-52	55-56	61-62
Females -	55-56	57-58	64-65

Comparisons may also be made of the numbers surviving at a few ages, according to the other Life-Tables.

Numbers surviving out of one million born.

Ages.	Haydock.	Brighton.	London.	Glasgow.	Manchester Township.
			MALES.		
1	864,905	849,990	836,078	825,310	769,260
5	781,289	751,250	722,999	668,700	623,260
15	749,435	725,010	691,698	617,990	575,400
35	690,244	640,900	604,990	521,480	443,150
55	510,539	453,030	410,293	340,610	206,800
75	143,897	166,660	124,399	87,110	29,060
			FEMALES.		
1	872,280	876,720	861,719	853,180	808,410
5	786,120	785,460	750,392	699,920	663,230
15	758,661	758,390	718,411	651,090	611,150
35	667,534	695,080	644,793	538,020	496.730
55	523,762	547,790	484,154	374,410	281,640
75	177,431	250,800	188,810	120.740	52,750

The last thing remaining to do, is to set side by side the numbers which it is the *final object* of a Life-Table to deduce; viz., those expressing in years the "mean after-life time," or "expectation of life," at the various ages. This may with advantage be done more fully than has been the case with the two preceding sets of comparisons.

Table showing comparison of the Expectation of Life, at various ages, as calculated by the Life-Table for Haydock, with the corresponding figures, *(a)* for England and Wales and *(b)* for the Selected Healthy Districts, together with the differences of the numbers for Haydock from the two Life-Tables for England respectively.

I.—MALES.

Age.	England and Wales.	Differences of Haydock from England and Wales.	Haydock.	Differences of Haydock from Healthy Districts.	Selected Healthy Districts.
0	43·66	+ 2·51	46·17	— 5·31	51·48
1	50·97	+ 1·37	52·34	— 5·05	57·39
2	53·04	+ 1·33	54·37	— 3·98	58·35
3	53·32	+ 1·47	54·49	— 3·63	58·12
4	53·15	+ 1·20	54·35	— 3·29	57·64
5	52·75	+ 1·06	53·81	— 3·24	57·05
10	49·00	+ 1·12	50·12	— 2·95	53·07
15	44·47	+ 1·44	45·91	— 2·71	48·62
20	40·27	+ 1·18	41·45	— 2·96	44·41
25	36·28	+ 0·72	37·00	— 3·39	40·39
30	32·52	+ 0·24	32·76	— 3·76	36·52
35	28·91	— 0·10	28·81	— 3·89	32·70
40	25·42	— 0·18	25·34	— 3·58	28·92
45	22·06	— 1·08	21·98	— 3·21	25·19
50	18·82	— 0·12	18·70	— 2·83	21·53
55	15·74	— 0·50	15·24	— 2·76	18·00
60	12·88	— 0·90	11·98	— 2·68	14·66
65	10·31	— 1·21	9·10	— 2·50	11·60
70	8·04	— 1·27	6·77	— 2·11	8·88
75	6·10	— 0·84	5·26	— 1·30	6·56
80	4·52	— 1·33	3·19	— 1·51	4·70
85	3·29	— 0·75	2·54	— 0·76	3·30

II.—FEMALES.

Age.	England and Wales.	Differences of Haydock from England and Wales.	Haydock.	Differences of Haydock from Healthy Districts.	Selected Healthy Districts.
0	47·18	— 0·23	46·95	— 7·09	54·04
1	53.24	— 0·47	52·77	— 5·80	58·57
2	55·18	— 0·58	54·60	— 4·76	59·36
3	55.46	— 0·41	55·05	— 4·06	59·11
4	55·31	— 0·29	55·02	— 3·60	58·62
5	54·92	— 0·51	54·41	— 3·60	58·01
10	51·10	— 0·73	50·37	— 3·64	54·01
15	46·55	— 0·35	46·20	— 3·48	49·68
20	42·42	— 0·31	42·11	— 3·51	45·62
25	38·50	— 0·14	38·36	— 3·36	41·71
30	34·76	— 0·20	34·56	— 3·35	37·91
35	31·16	— 0·18	30·98	— 3·18	34·16
40	27·60	— 0·65	26·95	— 3·43	30·38
45	24·05	— 0·98	23·07	— 3·49	26·56
50	20·56	— 0·80	19·76	— 2·99	22·75
55	17·23	— 0·93	16·30	— 2·76	19·06
60	14·10	— 1·10	13·00	— 2·56	15·56
65	11·26	— 1·35	9·91	— 2·45	12·36
70	8·77	— 1·61	7·06	— 2·47	9·53
75	6·88	— 1·33	5·35	— 1·79	7·14
80	5·00	— 0·73	4·27	— 0·96	5·23
85	3·71	— 0·65	3·06	— 0·71	3·77
90	2·95	— 0·12	2·63	— 0·08	2·71

As regards the mean "expectation of life" at Birth, the comparison may be made in another way—between Haydock and the whole of England and Wales.

	Males.	Females.	Differences of Females from Males.
England and Wales	43·66	47·18	+ 3·52
Haydock	46·17	46·95	+ 0·78
Differences of Haydock from England and Waies	+ 2·51	— 0·23	

These results, together with the "comparative mortality figures" in Table VI. as to the death-rates for Males and Females in the "Standard Population," show that the anomalous differences between the rates of mortality in Haydock during the 10 years as regards the two sexes, have been due chiefly to a *proportionately higher mortality among Females*, combined with (but to a much less extent) a *proportionately lower mortality among Males.*

Thus it will be noted that for *Males* in Haydock the "expectation of life" is greater than in England and Wales at all ages up to 30. Afterwards it becomes less.

For *Females* the "expectation of life" is a little less in Haydock than in England and Wales all down the Table to age 45, after which the difference becomes marked.

As regards both sexes, after age 60 the "expectation of life" becomes much less both for Males and Females. The same fact will appear from the future Table to be given showing the proportionate mean constitution of the population of Haydock, compared with the corresponding figures for England and Wales. See Table VII.

It may be also of interest to show comparisons between the "expectation of life" for Haydock at various ages, with the corresponding figures calculated in the other Local Life-Tables already referred to.

These are as follows:—

Ages.	Haydock.	Brighton.	London.	Glasgow.	Liverpool.	* Manchester Township.
MALES.						
0	46·17	43·59	40·66	35·18	34·3	28·78
5	53·81	52·87	50·77	46·97	45·9	40·53
10	50·12	49·12	47·22	44·32	42·9	37·47
15	45·91	44·67	42·88	40·51	38·8	33·56
25	37·00	36·51	34·70	33·29	31·0	26·00
35	28·81	29·02	27·39	26·06	24·2	20·01
45	21·98	22·36	21·00	19·54	18·3	14·93
55	15·24	16·48	15·31	13·39	13·2	10·96
65	9·10	10·96	10·59	9·38	8·8	7·48
75	5·26	6·64	7·20	5·96	—	4·74
FEMALES.						
0	46·95	49·00	44·91	37·70	36·4	32·67
5	54·41	56.92	54·42	48·27	47·7	43·66
10	50·37	53·15	50·95	45·44	44·7	40·94
15	46·20	49·07	46·65	41·59	40·6	37·05
25	38·36	40·48	38·34	34·60	32·7	29·41
35	30·98	32·48	30·69	28·06	25·9	22·90
45	23·07	25·07	23·80	21·61	19·8	17·20
55	16·30	18·48	17·34	15·60	14·3	12·25
65	9·91	12·19	11·78	10·69	9·8	8·54
75	5·35	6·97	7·55	6·97	—	6·03

* There are three separate Life-Tables for Manchester:—

No. 1—For Manchester City, including *all* the Townships of which it is made up. In this, the "expectation of life" at Birth is given as follows: Males 34·71, Females 38·44.

No. 2—For "Manchester Township" only—the worst part of the City as regards overcrowding and bad sanitary conditions.

No. 3—For Manchester "Outlying Townships." In this the "expectation of life" at Birth is given thus: Males 38·11, Females 41·58.

Another way in which the vital statistics of Haydock may be considered, is as to the comparison between these three particulars—

(1)–The Proportion of Number Dying to Number Living.

(2)–The Mean Age at Death.

(3)–The "Expectation of Life" at Birth.

It has been seen that as regards persons, that is males and females being taken together, the mean "expectation of life" at birth for 1881–90 has been 46·55 years. Now if the population of Haydock had been *stationary* for many preceding years, that is, the annual number of births just equalling the annual number of deaths, and no migration having occurred either of persons into Haydock or of persons out of Haydock, then the mean age at death, that is, the number obtained by adding up the separate exact ages of those who have died, and dividing the sum by the number dying, would have been exactly 46·55 years, and 1 in 46·55 persons would have died annually.

Whereas during the Decennium, the "mean age at death" has been only 25·3 years, and instead of 1 person in 46·55 dying annually, the proportion has been 1 in 58.

Now, these apparent contradictions are thus accounted for.

(1) The population of Haydock has been a greatly *increasing* one, that is, the number of births has exceeded the number of deaths every year. So that every year the average age of the dying, or the "mean age at death" has been greatly lowered by the large proportion of the deaths of young children.

This shows the fallacy of calculating a Life-Table, as has sometimes been done in past times, *from the ages of the dying only*, without taking into account the numbers and ages of the living. (A low mean age at death is no test at all of the healthiness of a district, it simply means that there is existing a high birth-rate).

(2) On the other hand, the fact of a continuous high birth-rate, every year adding to the population more than are taken from it by death and migration, comes in time to bring about the existence in the population of an excessive proportion of persons between the ages of 5 and 55, during which period the mortality is below the average, and therefore the proportionate mortality has been 1 in 58, instead of being 1 in 46·55.

Table comparing the vital statistics of Haydock with those of England and Wales for the 10 years, 1881-90, as regards (*a*) Proportionate Mortality, (*b*) Mean Age at Death, and (*c*) Mean "Expectation of Life" at Birth.

PROPORTIONATE MORTALITY.				MEAN AGE AT DEATH.			MEAN EXPECTATION OF LIFE AT BIRTH.		
	Males.	Females.	Persons.	Males.	Females.	Persons.	Males.	Females.	Persons.
HAYDOCK	1 in 59	1 in 56	1 in 58	25·7	24·9	25·3	46·17	46·95	46·55
ENGLAND and WALES	1 in 49	1 in 55	1 in 52	*30·5	*33·9	*32·1	43·66	47·18	45·39

*—These Figures are not given in the Registrar-General's Decennial Supplement, nor could they be obtained by application at the General Register Office. However, I have approximately worked them out myself from the total numbers given as dying at the various age-periods during the 10 years.

Allusion has been already made to the fact that the Life-Table now presented is the *third* which I have worked out to completion.

No. 1 was based (*a*) on the arithmetical means of the numbers enumerated at the Censuses of 1881 and 1891, without any attempt to exclude the Inmate population of Haydock Lodge, and (*b*) on the numbers of deaths registered during the ten years exactly corresponding to the Census interval, *i.e.*, from April 1st, 1881, to March 31st, 1891.

This was discarded:—

(1) Because of the *Population* errors.

- (*a*) The error of Arithmetical means.
- (*b*) The more serious error of including the Inmate Population of Haydock Lodge.
- (*c*) The error of taking the enumerated numbers for the first five years of life as correct.

(2) Because the *Number of Deaths* had not been *corrected* in any other way than by excluding the Inmate Deaths of the Asylum.

No. 2 was based (*a*) on the true mean *Total Population* of the 10 years 1881–90, and (*b*) on the numbers of Deaths in the 10 calendar years 1881-90, *corrected* in the manner previously indicated (the attempt to make the period for which the Deaths were taken *exactly* correspond with the Census interval was abandoned, as while one difficulty was thus got over, greater difficulties were introduced).

This Life-Table was rejected.

(1) Because while the *total* true mean Population had been correctly taken, the means for the separate age and sex groups had been calculated as at 5 years after the Census of 1881, instead of 4¾ years.

(2) Because the method of dealing with the Population numbers for the first five years of life, and of calculating the probabilities of survival for these years had not been quite correctly applied.

(3) Because it had not been worked out quite in the usual form; the survivors having only been calculated for each sex out of a million of each supposed to be born.

(4) Because the mean Population numbers at the even ages, 10, 20, 30, &c., had been simply deduced from the Census enumerations without correcting calculations.

No. 3 was worked out on the plan, and in the manner already fully described.

The results of these three Life-Tables may be thus compared, as to mean expectation of Life at Birth.

	MALES.	FEMALES.
No. 1 - - - -	45·10	47·21
No. 2 - - - -	45·93	46·96
No. 3 - - - -	46·17	46·95

It will thus be apparent that the results of the final Life-Table do not differ very materially from those of No. 2, and it may appear as if there had been no real need for having taken so much trouble. However, there is at least the satisfaction of having attempted to work out the results for a population of six thousands with as much accuracy as if the number had been six millions. And in working with Logarithms (pausing a moment to gratefully remember those by whose labour and ingenuity these Tables have been constructed), it really gives but little more trouble to take three "decimal places" instead of one.

The question may be asked—are all these elaborate Vital Statistics of any good at all? The answer must be "much, every way."

The progressive improvement in sanitary conditions during the last quarter of a century, with all its beneficent results in diminishing Disease and lengthening Life, was set going by the apparently dry and utterly forbidding columns of figures compiled by the late Dr. WILLIAM FARR, illuminated as they were by his lucid explanations and masterly grasp of the lessons to be deduced from them.

So, even the much less elaborate and more humble attempts of this Supplement, when attentively studied, may be lit up by grateful remembrance of all that has been involved in sanitary improvements already effected, and by the glow of looking forward to the further benefit to be derived by carrying out work yet remaining to be done in the future.

It should not be forgotten too, that the famous "Carlisle Life-Table" which was constructed about a century ago, by MILNE, from facts relating to the numbers living and dying in that city for the 9 years, 1779–87, supplied to him by Dr. HEYSHAM, was founded on a mean Population of only 8,177, and Deaths numbering 1,840. Therefore although for *small* populations there must necessarily be anomalies, and uneven fluctuations, which are eliminated by dealing with large populations, there may yet be interest and value more than what is merely local, in a Life-Table calculated for a population so small as 6,000, as demonstrating that, in spite of accidental variations, the *same general laws* are found to be prevailing, as those deduced from statistics relating to the whole country.

Apart from this, however, as soon as the time comes, some five or six years hence, when it may be possible to calculate another Life-Table for Haydock, founded on the mean Population and Deaths for the 10 years 1891–1900, the value of such will be greatly enchanced by the existence of the Life-Table now presented, as enabling comparisons to be made in the most exact and scientific manner possible between the Vital Statistics of the two Decennia.

In the changes and chances of life, it may not fall to my own lot to do this work, but knowing how great has been the labour of having to bring together the materials for a Life-Table, apart from the trouble of calculating it, it will at least be my endeavour to leave these materials for a future Life-Table compiled up to date.

I can only reiterate before I close, how great is the debt of gratitude which I owe to those who have taken such trouble to help me with the required statistics, and the much needed advice as to methods. In particular I must refer to Dr. TATHAM, to Mr. NOEL A. HUMPHREYS, and to Mr. A. C. WATERS of the General Register Office, who have taken such trouble as can only be accounted for by their being disposed, like Providence, to help those who at least *try* to help themselves. I should also not forget to mention that I am indebted to the respective Medical Officers of Health of Brighton, London, Manchester, Glasgow, and Liverpool for their courtesy in supplying me with copies of their respective Life-Tables, and in particular I have to express my appreciation of kindly interest and of advice rendered by Dr. A. NEWSHOLME, the M. O. H. of Brighton, and the author of one of the leading works on the subject of Vital Statistics.

It must be explained that I am alone responsible for the correctness of the figures and calculations, as I have had to undertake the work single-handed, and should experts find, as too well they may, flaws and errors, it may be remembered that this work has been done by one whose primary occupation is that of a busy General Practitioner of Medicine and Surgery, and who has had to do this work, not when he wanted, but when he could, and who in doing it has often had to rob himself of much needed rest and recreation.

It may be difficult for such experts, who have long since passed their preliminary difficulties, to put themselves in the place of a beginner. So it may appear to them, that this attempt to explain the construction of a Life-Table, which is, in the first instance, addressed to the members of my Sanitary Authority, and which it is hoped may not be without some advantage to other Medical Officers of Health who may be desirous of calculating Local Life-Tables for their Districts, may be very unnecessarily redundant and minute.

When we travel along a smooth and well-made road we are all too apt to forget that it has been made so by the long continued toil and labour of our predecessors. This at least, is the attempt of one who has repeatedly had the bitter experience of having to abandon the results of weeks of labour, to make the path easier for such as may desire to travel in the same direction.

In conclusion it may not be inappropriate to state that the *original stimulus* which led to the beginning of the work which has been finally embodied in this Supplementary Report, was conveyed to me, by one who had personally known the late Dr. WILLIAM FARR,* and who had derived from him a knowledge and love of Vital Statistics, and who some months ago by a few words of kindly yet keen criticism of my Annual Report for 1896, in which an attempt had been made to deal with the Sanitary History of Haydock during a period of 60 years, showed that however great was the *breadth* of my Report, it had attained to but little *depth*, and thus, with many stumblings and failures and mistakes, I have entered upon and carried through the work now completed.

Therefore these pages may be brought to an end with (shall it be said ?) reverent remembrance of the greatness of Dr. FARR as the Father of English Vital Statistics, and of his goodness as a benefactor of mankind, and should the writer be counted worthy of transmitting to others the impulse originally derived from him, the reward will be great.

* I refer to Mr. R. FARROW, of Leek, Staffordshire.

Preliminary Tables of Population and of Deaths giving the numerical facts on which the Life-Table for Haydock is based.

TABLE I.

Population numbers enumerated at the two Censuses of 1881, and 1891, (EXCLUSIVE of the Inmate Population of Haydock Lodge Lunatic Asylum), together with their "Arithmetical Means."

	MALES.			FEMALES.		
Ages.	Census 1881.	Census 1891.	Arithmetical Means.	Census 1881.	Census 1891.	Arithmetical Means.
0—5	480	504	492	501	491	496
5—15	777	837	807	731	844	787·5
15—25	553	760	656·5	389	530	459·5
25—35	469	535	502	405	357	381
35—45	309	368	338·5	239	312	275·5
45—55	221	211	216	183	182	182·5
55—65	118	161	139·5	121	147	134
65—75	59	67	63	65	72	68·5
75—85	13	13	13	14	15	14·5
85—95	1	0	0·5	0	2	1
95 upwards	0	0	0	0	1	0·5
Totals ...	3,000	3,456	3,228	2,648	2,953	2,800·5

TABLE II.

Calculated True Mean Population numbers in Age and Sex Groups for the 10 Calendar years 1881-90.

Ages.	MALES.	FEMALES.
0—5	491·992	497·925
5—15	805·820	783·270
15—25	646·598	452·788
25—35	499·645	384·616
35—45	336·092	272·128
45—55	217·158	183·057
55—65	137·451	132·897
65—75	62·720	68·292
75—85	13·036	14·483
85—95	0·558	0·690
95 upwards	0	0·445
Totals	3,211·070 or 3211	2,790·591 or 2791

TABLE III.

Table of Deaths for Haydock during the Ten Calendar Years 1881–90, classified according to Sexes and in Age-groups.

Ages.	MALES.				
	Deaths among the Inmates of Haydock Lodge Lunatic Asylum (excluded to begin with).	Deaths registered in Haydock (excluding the Inmates of the Asylum).	Deaths occurring in Haydock of 'non-residents' (to be deducted)	Deaths of Haydock residents in the Workhouse at Warrington or elsewhere in so far as is known (to be added).	Corrected Mean Annual Numbers of Deaths.
0–1	0	160	—	—	16·0
1–2	0	56	—	—	5·6
2–3	0	20	—	—	2·0
3–4	0	15	—	—	1·5
4–5	0	8	—	—	0·8
5–15	2	35	1	—	3·4
15–25	8	18	1	—	1·7
25–35	13	30	3	—	2·7
35–45	16	48	8	1	4·1
45–55	30	37	—	1	3·8
55–65	27	46	2	—	4·4
65–75	15	52	1	4	5·5
75–85	10	24	—	2	2·6
85–95	0	5	—	—	0·5
Totals	121	554	16	8	54·6
	FEMALES.				
0–1	0	147	1	1	14·7
1–2	0	53	—	—	5·3
2–3	0	26	—	—	2·6
3–4	0	17	—	—	1·7
4–5	0	8	1	—	0·7
5–15	0	28	—	—	2·8
15–25	3	23	—	—	2·3
25–35	10	29	—	—	2·9
35–45	14	22	1	—	2·1
45–55	11	29	—	1	3·0
55–65	22	37	—	—	3·7
65–75	20	49	—	—	4·9
75–85	9	26	—	1	2·7
85–95	1	3	—	—	0·3
Totals	90	497	3	3	49·7

ADDENDUM TO TABLE III.

(1) Number of Infants Dying *under* 6 months of age:—

Males - - -	114
Females - - -	96

(2) Mean Age at Death of the Infants Dying *under* 1 year of age:—

Males - - - -	3·8275 months.
Females - - - -	4·1950 ,,

TABLE IV.

Table of Male and Female Births, and of the Male and Female Deaths at ages 0—1, 1—2, 2—3, and 3—4, required for calculating the true mean Population in the first five years of Life for the Decennium, 1881—90.

BIRTHS.			DEATHS AT AGES							
			0—1		1—2		2—3		3—4	
Year.	Males.	Females.	M.	F.	M.	F.	M.	F.	M.	F.
1876	145	147	—	—	—	—	—	—	—	—
1877	119	118	12	7	—	—	—	—	—	—
1878	138	136	20	19	7	9	—	—	—	—
1879	135	147	23	18	6	7	2	5	—	—
1880	114	124	25	21	10	16	7	2	0	2
1881	121	120	17	16	5	2	2	2	0	3
1882	115	118	17	9	3	11	0	3	0	1
1883	132	137	14	13	4	2	2	0	3	2
1884	122	116	11	17	3	5	1	2	1	1
1885	118	121	13	15	15	7	5	8	3	3
1886	120	125	20	17	5	5	0	1	2	0
1887	130	126	18	14	9	7	4	3	3	1
1888	121	101	14	13	7	7	2	2	2	3
1889	122	107	23	13	3	4	2	3	1	1
1890	134	118	13	20	2	3	2	2	0	2

TABLE V. (PART 1).

Life-Table for Haydock based (a) on the true mean numbers LIVING, and (b) on the numbers DYING in the 10 years 1881-90.

MALES.

1	2	3	4	5	6	7
Age.	Number dying in each year or age-period of 5 years.	Number supposed to have been originally born, and number surviving at each age.	Mean population living in each year or age-period.	Years of life lived by the mean population in each year or age-period.	Sum of the years of life lived in each year or age-period and afterwards to the end of the Table.	Mean after-lifetime or "expectation of life" at each age. Years.
x	d_x	l_x	P_x		Q_x	$E_x = \frac{Q_x}{l_x}$
0	68,838	509,488	475,069	462.606	23,525,453	46·17
1	24.322	440,650	428,489	428,489	23,062,847	52·34
2	8,506	416,328	412,075	412,075	22,634,358	54·37
3	6,382	407,822	404,631	404,631	22,222,283	54·49
4	3,390	401,440	399,745	399,745	21,817,652	54·35
5	9,909	398,050	393,096	1,965,478	21,417,907	53·81
10	6,320	388,141	384,981	1,924,905	19,452,429	50·12
15	4,746	381,821	379,448	1,897,240	17,527,524	45·91
20	5,191	377,075	374,480	1,872,398	15,630,284	41·45
25	8,081	371,884	367,844	1,839,218	13,757,886	37·00
30	12,139	363,803	357,734	1,788,668	11,918,668	32·76
35	19,433	351,664	341,948	1,709,738	10,130,000	28·21
40	22,185	332,231	321,139	1,605,693	8,420,262	25·34
45	25,102	310,046	297,495	1,487,475	6,814,569	21·98
50	24,836	284,944	272,526	1,362,630	5.327,094	18·70
55	31,249	260,108	244,484	1,222,418	3,964,464	15·24
60	41,795	228,859	207,962	1,039,808	2,742,046	11·98
65	53,881	187,064	160,124	800,618	1,702,238	9·10
70	59,871	133,183	103,248	516,238	901.620	6·77
75	37,773	73,312	54,426	272,128	385,382	5·26
80	30,697	35,539	20,191	100,953	113,254	3·19
85	4.803	4.842	2,441	12,203	12,301	2·54
90	39	39	20	98	98	2·50
95		0				

TABLE V. (Part 2).

Life-Table for Haydock based (a) on the true mean numbers LIVING, and (b) on the numbers DYING in the 10 years 1881-90.

FEMALES.

1	2	3	4	5	6	7
Age.	Number dying in each year or age-period of 5 years.	Number supposed to have been originally born, and number surviving at each age.	Mean population living in each year or age-period.	Years of life lived by the mean population in each year or age-period.	Sum of the years of life lived in each year or age-period, and afterwards to the end of the table.	Mean after-lifetime or "expectation of life" at each age. Years.
x	d_x	l_x	P_x		Q_x	$E_x = \frac{Q_x}{l_x}$
0	62,648	490,512	459,188	449.765	23,027,178	46·95
1	22,013	427,864	416,858	416.858	22,577,413	52·77
2	10,573	405,851	400,565	400,565	22.160,555	54·60
3	6,875	395,278	391,841	391,841	21,759,990	55·05
4	2,802	388,403	387,002	387.002	21,368,149	55·02
5	6,991	385.601	382,101	1,910,528	20,981,147	54·41
10	6,478	378.610	375,371	1,876,855	19,070,619	50·37
15	7,566	372,132	368,349	1,841,745	17,193,764	46·20
20	11,152	364,566	358,990	1,794,950	15.352,019	42·11
25	11,476	353.414	347,676	1,738,380	13,557,069	38·36
30	14,505	341,938	334,686	1,673,428	11,818,689	34·56
35	10,696	327,433	322,085	1,610,425	10,145,261	30·98
40	13,946	316,737	309,764	1,548,820	8,534,836	26·95
45	22,959	302,791	291.312	1.456,558	6,986,016	23·07
50	22,920	279,832	268,372	1,341,860	5,529,458	19·76
55	28,151	256,912	242,837	1,214,183	4.187,598	16·30
60	35,203	228,761	211,160	1.055,798	2.973,415	13·00
65	43,523	193.558	171.797	858.983	1,917,617	9·91
70	63,003	150,035	118.534	592,668	1.058,634	7·06
75	50,351	87,032	61,857	309,283	465,966	5·35
80	25,003	36,681	24,180	120,898	156,683	4·27
85	10.396	11,678	6,480	32.400	35.785	3·06
90	1.246	1,282	659	3.295	3.385	2·63
95	36	36	18	90	90	2·50
100		0				

TABLE V. (Part 3).

Life-Table for Haydock, based (a) on the true mean numbers LIVING, and (b) on the numbers DYING, in the 10 years, 1881-90.

MALES AND FEMALES.

Age.	Chance (or probability) of living one year from each age. p_x		Of a Million Males born, the number surviving at each age.	Of a Million Females born, the number surviving at each age.	Of a Million of both sexes born, comprising 509,488 Males, and 490,512 Females.	
					The number surviving at each age.	Sum of the years of life lived in each year or age-period and afterwards to the end of the Table.
x	Males.	Females.	l_x	l_x	l_x	Q_x
0	·86491	·87228	1,000,000	1,000,000	1,000,000	46,552,631
1	·94481	·94855	864,905	872,280	878,514	45,640,260
2	·97957	·97395	817,165	827,404	822,179	44,794,913
3	·98435	·98261	800,469	805,848	803,100	43,982,273
4	·99155	·99279	787,944	791,831	789,843	43,185,801
5	·99497	·99635	781,289	786,120	783,651	42,399,054
10	·99672	·99655	761,840	771,867	766,151	38,523,048
15	·99750	·99590	749,435	758,661	753,953	34,721,288
20	·99723	·99380	740,121	743,235	741,641	30,982,303
25	·99562	·99347	729,931	720,500	725,298	27,314,955
30	·99324	·99137	714,071	697,104	705,741	23,737,357
35	·98870	·99338	690,244	667,534	679,097	20,275,261
40	·98627	·99103	652,101	645,727	648,968	16,955,098
45	·98326	·98435	608,555	617,295	612,837	13,800,585
50	·98193	·98305	559,287	570,490	564,776	10,856,552
55	·97473	·97706	510,539	523,762	539,940	8,152.062
60	·96047	·96713	449,203	466,371	457,620	5,715,461
65	·93431	·95033	367,170	394,601	380,622	3,619,855
70	·88745	·89680	261,411	305,875	283,218	1,960,254
75	·86518	·84130	143,897	177,431	160,344	851,348
80	·67122	·79539	69,756	74,781	72,220	269,937
85	·38119	·64286	9,504	23,807	16,520	48,086
90			77	2,614	1,321	3,483
95			0	74	36	90
100				0	0	.

TABLE VI.

Showing the calculation of the Haydock "Death-Rates in Standard Population" for Males, Females, and Persons.

(In this calculation the numbers of deaths are calculated in a million persons, divided up into age and sex groups in proportion to the mean age and sex distribution of the population of England and Wales during the decennium 1881-90, on the supposition that the death-rates in the several age and sex groups had been the same as those which have been calculated for Haydock during the same 10 years).

MALES.				FEMALES.		
Age.	Standard Population for England and Wales.	Death-Rates for Haydock.	Numbers of Deaths.	Standard Population for England and Wales.	Death-Rates for Haydock.	Numbers of Deaths.
0–5	64,122	52·643	3.375·590	64,557	50·208	3.241·301
5–10	59,333	5 648	335·093	59,673	5·021	299·608
10–15	54,806	2·478	135·829	54,765	1·735	95·217
15–20	49,720	2·597	129·138	50,287	4·730	237·847
20–25	42,922	2·666	114·427	47,564	5·549	259·182
25–35	71,131	5·404	384·380	77,499	7·540	584·342
35–45	55,095	12·199	672·106	58,944	7·717	454·868
45–55	40,472	17·499	708·211	44,478	16·388	728·921
55–65	27,151	32·011	869·142	30,893	27·841	860·095
65–75	15,184	87·691	1,331·505	18,326	71·751	1,314·904
75 upwards	5,591	228·044	1,274·982	7,487	192·086	1,438·150
TOTAL Male Populat'n.	485,527	TOTAL Male Deaths	9,330·403	514,473		9,514·435
TOTAL Female Populat'n	514,473	TOTAL Female Deaths	9.514·435			
	1,000,000		18,844·838			

Death-Rates in "Standard Population."

MALES. $\frac{9.330·403}{485,527} \times 1.000 = 19·22$

FEMALES. $\frac{9.514·435}{514.473} \times 1,000$ 18·49

PERSONS. $\frac{18,844·836}{1,000,000} \times 1,000 = 18·84$

TABLE VI.—continued.

Comparisons of Mean Annual Death-Rates for 1881-90.

	England * and Wales.	Haydock.		
		Crude Death-Rates.	Death-Rates in "Standard Population."	Comparative Mortality Figures—England and Wales = 1,000.
Males	20·29	17·00	19·22	947
Females ...	18·08	17·81	18·49	1.023
Persons ...	19·15	17·38	18·84	984

* The numbers as given in the Registrar General's Decennial Supplement, Part I., are respectively :—

Males	20·22
Females	18·01
Persons	19·08

These rates, however, have been calculated on the "Arithmetical Means" of the Census enumerations of 1881 and 1891. In order to make them comparable with the rates as they have been worked out for Haydock, on the *true* mean population numbers, they require to be multiplied by the proper "Factor of Correction" which = 1·0036686.

TABLE VII.

Showing the distribution of the mean population of Haydock, in age and sex groups, in proportions per million, for the 10 years, 1881-90, compared with the corresponding figures for England and Wales.

Ages.	MALES.		FEMALES.		PERSONS.	
	Haydock.	* England and Wales.	Haydock	* England and Wales.	Haydock.	England and Wales.
0—5	81,971	64,122	82,960	64,557	164,931	128,679
5—10	73,752	59,333	73,005	59,673	146,757	119,006
10—15	60,506	54,806	57,497	54,765	118,003	109,571
15—20	57,733	49,720	38,749	50,287	96,482	100,007
20—25	49,997	42,922	36,691	47,564	86,688	90,486
25—35	83,246	71,131	64,105	77,499	147,351	148,630
35—45	55,997	55,095	45,340	58,944	101,337	114,039
45—55	36,181	40,472	30,499	44,478	66,680	84.950
55—65	22,901	27.151	22,142	30,893	45,043	58,044
65—75	10,450	15,184	11,378	18,326	21.828	33,510
75 upwards	2,265	5,591	2,635	7,487	4,900	13,078
All ages ...	534,999	485,527	465,001	514,473	1,000,000	1,000,000
	465,001	514,473				
	1,000,000	1,000,000				

*—The numbers for England and Wales, which have been taken from the Registrar-General's Decennial Supplement, Part I., would appear to have been calculated from the "Arithmetical Means" of the two Census enumerations of 1881, and 1891, and therefore to be not strictly comparable with the numbers for Haydock, which have been calculated from the true mean population numbers, but the differences will not be sufficiently great to invalidate the comparisons made.

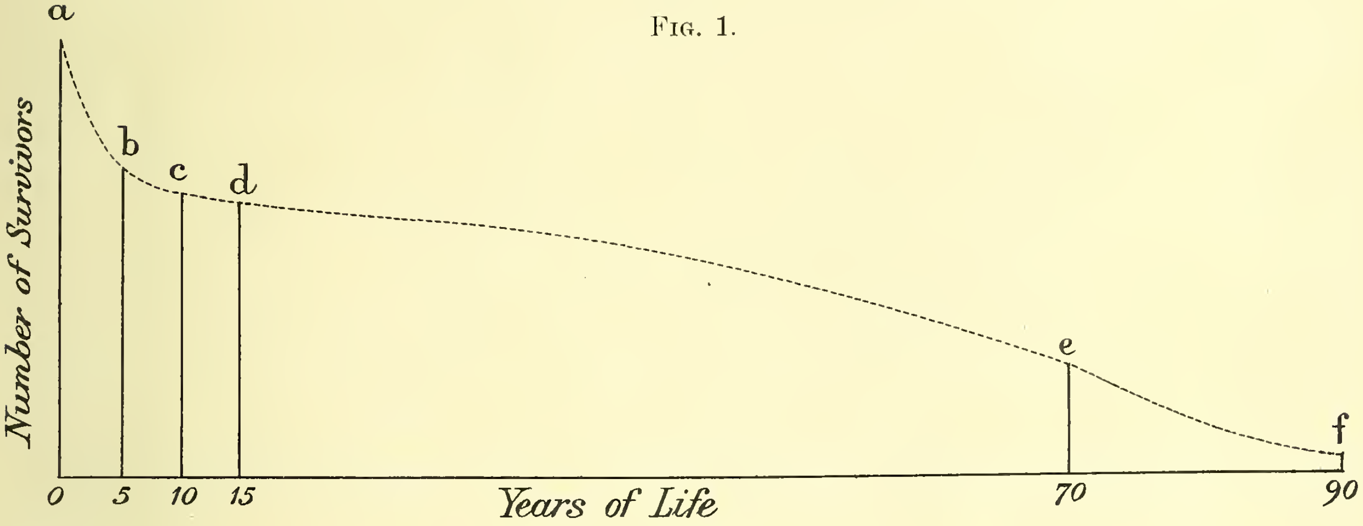
Geometrical explanation, in brief outline, of the problem involved in calculating the years of life lived in quinquennial periods from the data—

(1) Number of survivors "l_x," at the beginning of a 5-yearly period.

(2) Number of survivors "l_{x+5}," at the end of the period.

If the numbers of survivors at the end of each separate year in a complete Life-Table be shown "graphically" by placing a series of dots arranged to scale with reference to two lines, (a) a horizontal one to represent the years of life and (b), a vertical one to represent the numbers of survivors, and then a continuous line be drawn through these dots, a "curve" will be obtained something like that shown below. (This is not, however, drawn to scale, but is merely intended to give a rough general idea).

FIG. 1.



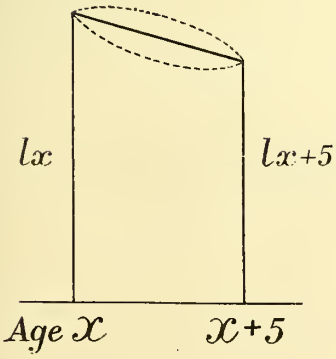
Broadly and generally speaking this Life-Table curve may be divided into four parts.

(1) A deeply concave part from *a* to *c*, corresponding to the first 10 years of life. During this time the mortality is *decreasing*, at first very rapidly from *a* to *b*, that is from age 0 to age 5, and then less rapidly from *b* to *c*, corresponding to the period between age 5 and age 10.

(2) From *c* to *d*, corresponding to the age-period 10–15. This is but little curved and forms a slightly descending nearly straight line. The mortality before the end of this period has begun to increase a little.

(3) From *d* to *e*, corresponding to the years of life from 15 to about 70. This forms a descending curve decidedly convex upwards. During this time the rate of mortality is increasing more and more.

FIG. 2



(4) From *e* to *f*. During this period from about age 70 onwards, the curve again becomes slightly concave upwards, and the rate of mortality is very rapidly increasing.

Now the total years of life lived through by a population of l_x, decreasing in the 5 years between age x and age $x+5$ to l_{x+5}, are to be represented geometrically by the area bounded below by the line x to $x+5$, and at the sides by the lines l_x and l_{x+5}, and above by the intercepted portion of the Life-Table curve—which may be (1) concave, (2) approximating somewhat to a straight line, or (3) convex (see Fig. 2).

If the Death-rate be *rapidly decreasing* the curve will be deeply concave.

If the Death-rate (or "force of mortality") be *constant* during the 5 years, the curve will be a slightly concave one called "the Logarithmic curve" and the years of life lived in the 5 years can be *exactly* calculated by the formula $$5 \times \frac{l_x - l_{x+5}}{Hyp.\ Log.\ \frac{l_x}{l_{x+5}}}$$

At no part of the Life-Table curve, however, can it be said that the "force of mortality" remains constant for 5 years.

Anything less concave than the Logarithmic curve, that is more nearly approaching to a straight line, and still more so, a convex curve, will represent an *increasing* mortality.

Taking the arithmetical mean of l_x and l_{x+5}, multiplied by 5, as giving the years of life lived during the 5 years is equivalent to assuming that the "curve" is a *straight line*, and that the deaths are evenly distributed through the 5 years, so that an equal number will be occurring in each of the 5 years.

If the curve be deeply concave the arithmetical mean will give a value decidedly too great—but during the age-period 0—5 to which this consideration would most strongly apply, the years of life lived have been calculated for the separate years. It also applies, however, to the age-period of 5—10.

If the curve be only a little concave, *i.e.*, rather less concave than the Logarithmic curve (or, in other words, more nearly approaching to a straight line), the arithmetical mean will give a value only a little too great. This may apply, more or less, to the part of the Life-Table curve from age 70 onwards. And if the curve be *convex* upwards, *as is the case during the greatest part of the Life-Table curve,* then the arithmetical mean will give a value rather *less* than the true mean.

To take the "geometrical mean" of l_x and l_{x+5} would be simply to assume that there is one particular kind of concave curve, and, as has been seen, this would be inaccurate for the greater part of the Life-Table curve. Further, when applied to this calculation, the geometrical mean would imply a *higher* rate of mortality at age 40 than at age 41, and at age 41 than at age 42, &c., which is absurd.

On the other hand, the arithmetical mean implies an *increasing* rate of mortality during each of the 5 years of a given quinquennium, which is in accordance with the actual facts, at least after age 15. Thus to take a particular instance, in the Haydock Life-Table for Males we have 332,231 men at age 40, reduced to 310,046 at age 45, by 22,185 deaths in the 5 years. These deaths are assumed to be equally distributed over the whole period, that is, 4,437 will happen in each year. Thus, in the year of life following age 40, 4,437 deaths will occur out of 332,231 living, in the year of life following age 41, 4,437 deaths will occur out of 327,794 living, in the next, year 4,437 out of 323,357, in the next, 4,437 out of 318,920, and in the last, 4,437 out of 314,483, thus giving every year a *greater proportionate mortality.*

The following figures represent the actual working out of the results for this age-period by the respective methods of Arithmetical, Logarithmic, and Geometrical means.

Referring to the following Diagram (Fig. 3) (which may be used for illustration, althought not drawn for *this* calculation), we have given two upright lines, or "ordinates," $l_{40} = 332{,}231$, and $l_{45} = 310{,}046$, erected upon the horizontal line, or "abscissa," at a distance apart representing 5 years. The ordinates will be joined at their upper extremities by some sort of curve. The area enclosed will depend upon the shape of the curve, and has to be estimated by the mean height of the ordinates.

(1) If the "curve" be a straight line, the mean height of ordinates will be $\frac{l_{40}+l_{45}}{2}$ This is the Arithmetical mean or "A."

(2) If the curve be the "logarithmic curve," the *actual mean height* of the ordinates will be found by the formula $\dfrac{l_{40}-l_{45}}{Hyp.\ Log.\ \dfrac{l_{40}}{l_{45}}}$ This may be called the Logarithmic mean or "L."

(3) If the curve be the one corresponding to the Geometrical mean, the mean height of the ordinates may be taken as $\sqrt{l_{40}\times l_{45}}$ This may be called "G."

If A, L, and G be compared with each other, it will be found (very nearly) that A is longer than L by twice the distance that L is longer than G.

Therefore $L = \dfrac{A + 2G}{3}$, and $G = \dfrac{3L - A}{2}$

In this case
$A = 321{,}138{\cdot}5$
$L = 321{,}010{\cdot}8$
$G = 320{,}947$

and the years of life lived are to be deduced from each of these values by multiplying by 5.

Therefore the years of life lived during the age-period 40—45, are as follows:—

By Arithmetical mean = 1,605,693
By Logarithmic mean = 1,605,054
By Geometrical mean = 1,604,735

In order to calculate the values of the intermediate yearly ordinates corresponding respectively to A, L, and G, the methods indicated under the respective headings below are to be adopted, and thus the yearly rates of mortality assumed by the different "means" may be worked out.

Annual Rates of Mortality assumed by Arithmetical mean.

The numbers of survivors at each separate year of the 5-yearly period are to be obtained by successive subtractions of $\dfrac{332{,}231 - 310{,}046}{5} = 4{,}437$

	No. of Survivors.	No. of Deaths.	Death-rate per 1,000
Age 40	332,231	4,437	13·36
„ 41	327,794	do.	13·54
„ 42	323,357	do.	13·72
„ 43	318,920	do.	13·91
„ 44	314,483	do.	14·11
„ 45	310,046		

Annual Rates of Mortality assumed by Logarithmic mean.

We have 332, 231 decreasing in 5 years to 310,046,

therefore Annual rate of Decrease $= \sqrt[5]{\dfrac{310{,}046}{332{,}231}}$

$$\text{Log. Annual rate} = \frac{\text{Log. } 310{,}046 - \text{Log. } 332{,}231}{5} = \frac{5{\cdot}4914261 - 5{\cdot}5214401}{5} = \bar{1}{\cdot}9939972$$

Therefore Annual rate = ·98627

	No. of Survivors.	No. of Deaths.	Death-rate per 1,000
Age 40	332,231	4,561	13·73
„ 41	327,670	4,497	do.
„ 42	323,173	4,437	do.
„ 43	318,736	4,375	do.
„ 44	314,361	4,315	do.
„ 45	310,046		

Annual Rates of Mortality assumed by Geometrical mean.

(Having given above, (*a*) the number of survivors at each year, as deduced by the Arithmetical mean, which may be called "A," and (*b*), the corresponding numbers deduced by the Logarithmic curve, which may be called "L," the corresponding yearly numbers for the Geometrical mean, which may be called "G," are to be very nearly deduced as follows:— Since (very nearly) $L = \frac{A + 2G}{3}$ therefore $G = \frac{3L - A}{2}$)

	No. of Survivors.	No. of Deaths.	Death-rate per 1,000
Age 40	332,231	4,623	13·92
,, 41	327,608	4,527	13·82
,, 42	323,081	4,437	13·73
,, 43	318,644	4,344	13·63
,, 44	314,300	4,254	13·53
,, 45	310,046		

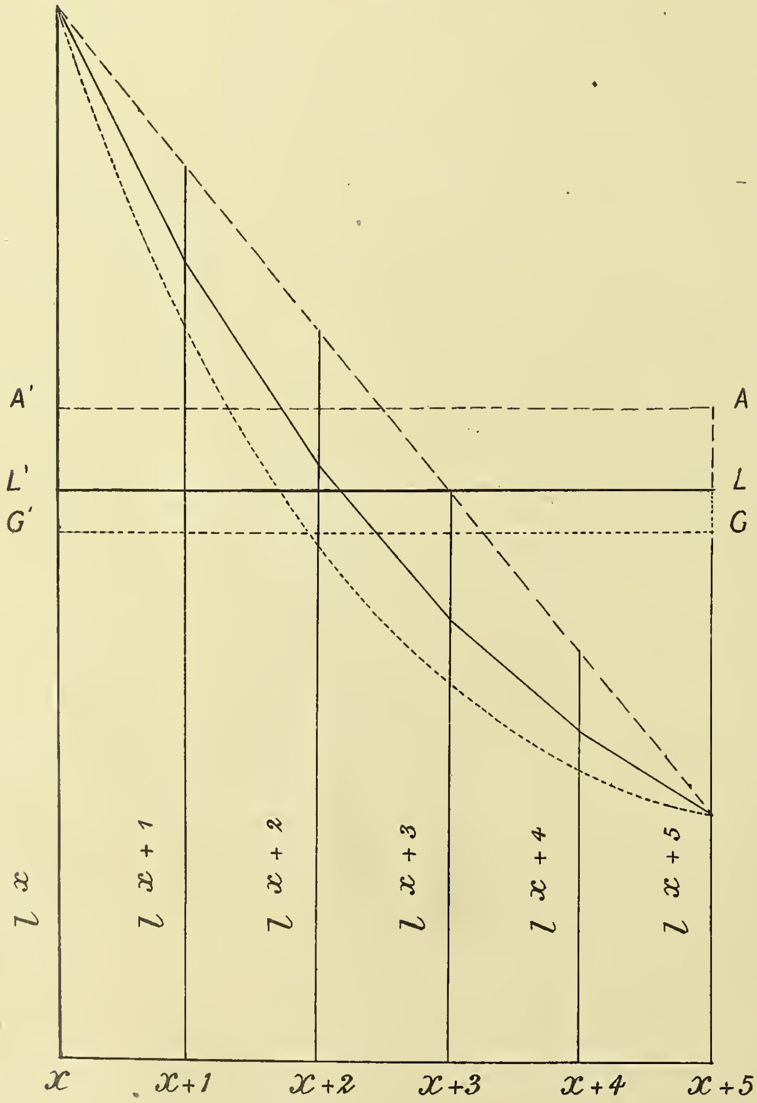
All, however, that is shown by the above figures is the relation which these three means, A, L, and G, bear to each other. But, from the facts—

(1) That the Arithmetical mean implies an increasing rate of mortality, which is in accordance with what actually exists, and

(2) That from the shape of the Life-Table curve the true years of life will be even greater in number than the value deduced by A, it is at least evident that A will give a nearer approximation to the truth than either L or G.

DIAGRAM TO ILLUSTRATE THE GRAPHIC CONSTRUCTION OF THE ARITHMETICAL, GEOMETRICAL, AND LOGARITHMIC MEANS OF l_x AND l_{x+5}.

FIG. 3



In order, if possible, to make the matter clearer, the argument may be re-stated in the following way, to illustrate which the Diagram (Fig. 3) has been specially drawn.

Let an *extreme* case be taken to illustrate the calculation of the years of life lived between age x and age $x+5$.

Suppose the Life-Table to show that the survivors are reduced to rather less than one-fourth as many at age $x+5$ as at age x. This state of things might be reached by a constant reduction of one-fourth in each intermediate year, that is, the chance of living one year or "p"$=\cdot75$—so that $p_x = p_{x+1} = p_{x+2} = p_{x+3} = p_{x+4} = \cdot75$.

If this were the law of survival then the years of life lived would be *overstated* by the arithmetical mean, or $\frac{l_x + l_{x+5}}{2} \times 5$, and *understated* by the geometrical mean, that is, by $\sqrt{l_x \times l_{x+5}} \times 5$.

The *true* value of the years of life lived would be found by the formula $\frac{l_x - l_{x+5}}{Hyp. Log. \frac{l_x}{l_{x+5}}} \times 5$ or, calling the arithmetical mean "A" and the geometrical mean "G," the value deduced from the last formula almost exactly corresponds to $\frac{A+2G}{3}$.

In the Diagram (Fig. 3) the arithmetical mean value of the years of life lived is represented by the parallelogram A^1, x, x + 5, A. , the geometrical mean by the parallelogram G^1, x, x + 5, G. , and the logarithmic mean by the parallelogram L^1, x, x + 5, L.

It should be noted that the distance of G to L should be exactly one-third of the distance of G to A.

The middle "curve" in the Diagram is intended to represent the "curve of equal proportional decrements," or the "Logarithmic" curve.

By the hypothesis with which we started, viz., that $p = \cdot75$, the value of l_{x+1} would be three-fourths the value of l_x, the value of l_{x+2}, three-fourths that of l_{x+1}, and so on. The curve has therefore been constructed by marking off points in the upright lines, or "ordinates," so that each of these ordinates shall be $\frac{3}{4}$ of the previous one, and then straight lines have been drawn from one point to the next—(This makes the construction clear, and every curve may be conceived of as consisting of an infinite number of straight lines).

Again, if the law of survival were that *equal numbers* died in each year, the chance of living, or p_x, would decrease from about $\cdot85$ in the first year to about $\cdot61$ in the fifth year, and $\frac{l_x + l_{x+5}}{2} \times 5$. would give the true years of life.

In the Diagram, as has been already explained, the "curve" of the arithmetical mean is represented by drawing a straight line joining the extreme ordinates.

The lower dotted curved line is intended to approximately represent the curve of the geometrical mean.* Each of the yearly ordinates is less than the corresponding ordinate of the Logarithmic curve by a length equal to (very nearly) half the difference of the Arithmetical and Logarithmic ordinates.

The geometrical mean would therefore give true results if the Life-Table curve were something like this, which it is NOT. Further, the formula $\sqrt{l_x \times l_{x+5}} \times 5$ would correctly represent the years of life only if the probability of living a year were *less* (and therefore the rate of mortality *greater*) at the beginning than at the end of the 5-year period. Thus, in the instance now being considered, p_x would be about $\cdot70$, and p_{x+5} would be about $\cdot85$. (This has been previously demonstrated by the actual example worked out).

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*—This curve might have also been represented by a series of straight dotted lines joining the intersected points in the ordinates.

As a matter of fact, at all periods after early childhood, the probability of living a year becomes *less* (and the rate of mortality *greater*) from year to year as age goes on.

The Life Table curve, therefore, generally lies on the *same* side of the curve of equal proportional decrements (Logarithmic curve), as does the straight line joining the ends of the section of curve under consideration—when it is concave, it lies between the Logarithmic curve and the straight line, and when it is convex it lies beyond, or outside, the straight line.

There is, therefore, an *a priori* probability that $\frac{l_x + l_{x+5}}{2} \times 5$ gives the years of life more correctly than does $\sqrt{l_x \times l_{x+5}} \times 5$.

This probability is strengthened by a couple of cases taken haphazard from the most recent Life-Table for England and Wales (Males), which, of course, has been worked out in the "extended" form, that is, for each year of life *separately*.

I.—TAKE FIRST OF ALL THE AGE-PERIOD 35–40.

Number of Males surviving at age 35 (or l_{35}) = 325,694.

Number of Males surviving at age 40 (or l_{40}) = 308,015.

(The numbers for the intermediate years, that is, l_{36}, l_{37}, l_{38}, l_{39}, need not be quoted.)

The TRUE years of life lived during these 5 years are to be deduced from the Life-Table by subtracting the YEARS OF LIFE LIVED IN AND ABOVE AGE 40 (or Q_{40}) from THE YEARS OF LIFE LIVED IN AND ABOVE AGE 35 (or Q_{35}). Now $Q_{35} - Q_{40} = 9{,}414{,}979 - 7{,}829{,}599 = 1{,}585{,}380 =$ true years of life $= \frac{l_{35} + 2\,(l_{36} + l_{37} + l_{38} + l_{39}) + l_{40}}{2}$

The arithmetical mean, that is $\frac{l_{35} + l_{40}}{2} \times 5 = 1{,}584{,}273$.

The geometrical mean, that is $\sqrt{l_{35} \times l_{40}} \times 5 = 1{,}583{,}650$.

Therefore—

the error of arithmetical mean = —1,107 (or 1 in 1,432).
the error of geometrical mean = —1,730 (or 1 in 916).

II.—SIMILARLY LET THE AGE-PERIOD 65–70 BE TAKEN.

$l_{65} = 164{,}202$. $l_{70} = 121{,}507$.

$Q_{65} - Q_{70} = 1{,}692{,}398 - 977{,}076 = 715{,}322 =$ true years of life $= \frac{l_{65} + 2\,(l_{66} + l_{67} + l_{68} + l_{69}) + l_{70}}{2}$

$\frac{l_{65} + l_{70}}{2} \times 5 = 714{,}273$, and $\sqrt{l_{65} \times l_{70}} \times 5 = 706{,}252$.

Error of arithmetical mean = —1,049 (or 1 in 682).

Error of geometrical mean = —9,070 (or 1 in 79).

This makes it reasonably certain that the arithmetical mean gives results which are not only more correct than those of the geometrical mean, but which are themselves VERY NEAR INDEED TO THE TRUTH. For, it is evident that if, in the English Life-Table we had had only l_{35} and l_{40}, or l_{65} and l_{70}, to work from, a much nearer approximation to the true value of $Q_{35} - Q_{40}$, or of $Q_{65} - Q_{70}$, would have been obtained by taking the Arithmetical means, than either the "Logarithmic" or Geometrical means. Therefore, in the Haydock Life-Table, having only the numbers l_x and l_{x+5} to work from, it has been best to use the Arithmetical means, as, thus, a truer result has been obtained than could have been arrived at by any method except one much too elaborate and complicated to be more than alluded to.

It may here be incidentally mentioned that the second of the two earlier discarded Life-Tables which I worked out was calculated both for arithmetical and geometrical means. The following were the comparative results obtained as regards the mean "expectation of life" at Birth:—

	MALES.	FEMALES.
By Arithmetical means	45·93	46·96
By Geometrical means	45·70	46·75

At the later age-periods, the differences, of course, became very marked.

Explanation of the apparent paradox that the last part of the Life-Table curve is *concave*, although the *mortality is rapidly increasing.*

(1) The chance of living is not measured by the slope of the Life-Table curve, but by the ratio of successive ordinates.

(2) The slope of the curve depends (*a*) on the ratio of the ordinates and (*b*) on their distances apart.

(3) In a Life-Table curve the successive decrements of the ordinates are the *numbers* of deaths. These, of course, tend to *increase* with a greater rate of mortality, but to *decrease* with decreased numbers of survivors.

$$\frac{\text{Decrement of ordinates}}{\text{Mean of ordinates}} = \text{Mean rate of mortality.}$$

For the line of reasoning in the preceding argument, and to a very large extent for the mode of expressing the ideas and mathematical formulæ involved, I am indebted to help which Mr. A. C. WATERS has been good enough to give me from time to time in answering questions as to difficulties and in writing explanatory notes thereon. In arranging the order of what has been said, and in, perhaps needlessly, repeating sometimes the same thing again and again in different ways, I have had the object in view of endeavouring to make the matter as clear to others, who may have even less than my own little mathematical knowledge, as he has made it clear to me. In this, only too probably, I may have but very indifferently succeeded. However, in thus fully and frankly acknowledging the source of my help, I would wish to carefully guard against any possibility of Mr. WATERS seeming to be in any degree responsible for my own errors in failing either to understand his reasoning or in the attempt to interpret it to others.

For some things, such as the use of the expression "Logarithmic mean," and the mode of attempting the graphic construction of the curve corresponding to the geometrical mean, I have to take the responsibility myself.

ADDENDUM.

Since the preceding pages were put into type it has occurred to me that one thing more remains to be done to make the work which I have undertaken complete.

A Life-Table has been worked out for Haydock which only professes to be an APPROXIMATION to the results which would be obtained by constructing an "Extended" Life-Table (that is, one calculated for every separate year of life).

It is of importance, therefore, to determine, if possible, how nearly the results obtained by the abbreviated method used (which has been fully described) can come to the perfect results of an extended Life-Table.

This determination might be made in one or other of two ways.

(1) By working out an extended Life-Table for Haydock and comparing the results, or

(2) By working out an abbreviated Life-Table for 5–yearly periods for England and Wales from the same data as those from which the last complete Life-Table was calculated.

The latter method is for many reasons the best to adopt.

In the Supplement to the 35th Annual Report of the Registrar-General, the late Dr. **FARR** gave the results of a Life-Table for England and Wales calculated by a "short" method which he had devised, from the same data as those from which his English Life-Table No. 3 had been worked out, viz., the numbers LIVING and DYING in England and Wales during the 17 years 1838-54, and these results were brought into comparison with those of the extended Life-Table.

The following is a copy of the Tabular Statement. (The "extended" Life-Table is denoted by "A" and the "short" Life-Table by "C").

AGES.	MEAN AFTER-LIFETIME.		
	By "A"	By "C"	Differences of "C" from "A"
0	39·91	40·00	+ ·09
5	49·71	49·82	+ ·11
10	47·05	47·05	± ·00
15	43·18	43·21	+ ·03
25	36·12	36·49	+ ·37
35	29·40	29·80	+ ·40
45	22·76	23·19	+ ·43
55	16·45	16·89	+ ·44
65	10·82	11·35	+ ·53
75	6·49	7·39	+ ·90
85	3·73	5·48	+ 1·75
95	2·17	5·00	+ 2·83

In this "short" method, the calculations for the first five years of life were taken as already made in the extended Life-Table. Afterwards the mean chances of living one year were worked out from the numbers of the living and of the dying for the 5-yearly age-periods 5–10 and 10–15, and then for successive 10-yearly periods.

Since this, I am not aware that any attempt has been made to compare the results to be obtained by this short method with those of the subsequent complete Life-Tables for England and Wales.

It therefore seemed desirable to show how the somewhat more elaborate short method which has been described as applied to the Haydock Life-Table would compare in its results, when applied to England and Wales, with the last extended Life-Table for England and Wales.

The data provided are those corresponding to what have been given for Haydock in the preceding Tables I., II., III., and IV.

For the first 5 years of life the calculations have been taken as already made in the extended Life-Table for England and Wales given in the Supplement to the 55th Annual Report of the Registrar-General, Part I. The true mean Population numbers and the numbers of deaths at the various age-periods are given in Part II. of the same Supplement, Table XI. These numbers have been dealt with in the same way as has already been described (see pages 19–21), by interpolating intermediate terms in the series of Logarithms corresponding to the numbers representing u_{15}, u_{25}, u_{35}, &c., both for Population $-\frac{1}{2}$ Deaths and Population $+\frac{1}{2}$ Deaths, and then from the intercepted numbers corresponding to the 5-yearly age-periods obtaining the mean chances of living one year by $\frac{P-\frac{1}{2}d}{P+\frac{1}{2}d}$.

It should be noted, however, that for the age-periods 5–10, and 10–15, the numbers of Population and Deaths have been taken as given in the Table XI. referred to, as these were found to give results *nearer* to those of the extended Life-Table than those obtained by the formula for interpolating u_{10}.

A difficulty has been met with in completing the abbreviated Life-Table after age 85, as the figures given in the Table XI. referred to do not extend any further than AGE 85 AND UPWARDS.

It has been impossible therefore to interpolate u_{90}. Under these circumstances the only way to get over the difficulty seemed to be to allow for the 15,917 survivors REMAINING at age 85 (l_{85}), years of after-lifetime in proportion to those calculated for the 15,207 survivors at the same age (l_{85}) in the extended Life-Table.

This procedure is, of course, open to the objection that the calculated mean after-lifetime at the later ages is made *less* than it would have been by continuing the calculation on the same lines.

Still, a study of the following Table VIII. will show that by the method employed a "short" Life-Table can be constructed for 5-yearly periods so much more closely approximating to the results of an extended Life-Table, on the whole, than does a short Life-Table calculated by the usual easier and "cruder" method, as to make it quite worth while, if a local Life-Table is to be worked out at all, to devote to it the little extra labour and trouble involved in the method of interpolation.

Since writing the above I have worked out two other "short" Life-Tables for England and Wales (males), the results of which are given in Table IX.

(1) The first of these called C in the Table was constructed by the simple short method of Dr. FARR. It had however to be carried on after age 85 by taking the results of the extended Life-Table, and therefore the differences in excess do not come out so strongly marked as those in Dr. FARR's own short Life-Table. Still in so far as it goes the results are parallel—a very near approximation at the earlier ages, and increasing differences in the way of excess at the later ages.

(2) The remaining Life-Table (called D in Table IX.), was worked out from an idea which may be considered my own, at least in so far that it occurred to me without being consciously directed to it by any one else.

A reference to pages 17–18 will show the considerations on which the idea was based.

It seemed to me that even if the mean chances of living one year be calculated for 10-yearly periods from the numbers of Population and Deaths at the respective periods, without interpolation, a nearer approach to the results of a perfect Life-Table would probably be obtained by making the calculations for each 10-yearly period AT TWO STAGES INSTEAD OF ONE.

Thus to take an actual example from the Life-Table D:—

At age 65 the number of survivors or (l_{65}) is 166,542. The mean chance of living one year in the interval from age 65 to age 75 has been calculated from the numbers of Population and Deaths, as $\frac{P - \frac{1}{2}d}{P + \frac{1}{2}d} = \frac{415{,}418 - 14{,}644}{415{,}418 + 14{,}644} = \cdot 93190$.

In the Life-Table C the number of survivors at age 75 (or l_{75}) was thus arrived at, $166{,}542 \times (\cdot 93190)^{10} = 82{,}264$.

The mean number living in the interval from age 65 to age 75 (or P_{65}) $= \frac{166{,}542 + 82{,}264}{2} = 124{,}403$, and the years of life lived $= 124{,}403 \times 10 = 1{,}244{,}030$.

On the other hand in Life-Table D the calculation was made thus:—

$$(1)\ 166{,}542 \times (\cdot 93190)^5 = 117{,}048.$$
$$(2)\ 117{,}048 \times (\cdot 93190)^5 = 82{,}264.$$

mean number living from age 65 to age 75 (or P_{65})

$$= \frac{\frac{166{,}542 + 117{,}048}{2} + \frac{117{,}048 + 82{,}264}{2}}{2} = 120{,}725 \cdot 5$$

and the years of life lived $= 120{,}725 \cdot 5 \times 10 = 1{,}207{,}255$

This last number therefore is found to differ from the number found by the ONE STAGE calculation by — 36,775.

A reference to Table IX. will show that by this method a REMARKABLY ACCURATE result has been obtained at the later ages, whereas as might have been expected, at the earlier ages there is an error in the way of deficiency.

The two Life-Tables C and D will be seen to MUTUALLY COMPLEMENT each other: C gives nearly true results at the ages 0, 5, and 10, and D nearly true results at the later ages.

If it be desired therefore to merely work out the mean expectation of Life for 10-yearly intervals after age 15, the combination of these two methods C and D appears to give promise of a very near approximation to the truth.

Both Life-Tables, by method C, and by method D, would have to be worked out to completion (at least in so far as to arrive at the years of life lived in each age-period). Up to age 15 these years of life would be identical in both Life-Tables. Then for the Q_x and E_x columns, the results of C would be taken for ages 0, 5, and 10, the rest being discarded, and the continuation would have to be made by D.*

It must be noted, however, that method D DOES NOT GIVE ACCURATE RESULTS FOR THE EVEN INTERMEDIATE AGES 40, 50, 60, &c.

These differ *by excess* from the results of the extended Life-Table.

The intermediate differences are indicated in Table IX.

While admitting these advantages of the combined method of C and D, the method of interpolation, B has the following considerations in its favour.

(1) All throughout from the first to the last of the quinquennial periods it gives an even and fairly close approximation to the results of A.

(2) For local Life-Tables it would tend to more evenly distribute anomalies and accidental variations, and generally to make a *smoother* Life-Table.

In conclusion it must be understood that, apart from the work of the English Life-Table which I have taken as already calculated for the first five years of Life, and after age 85, the figures presented are given solely on my own responsibility.

All possible effort has been made to be accurate, and it can only be hoped that, should anyone take the trouble to check the calculations, no very serious errors will be found.

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*—As will be afterwards pointed out, the truest result for age 10 is to be obtained by taking the *mean* of the results arrived at by methods "C" and "D."

TABLE VIII.

Showing the results obtained by working out a LIFE-TABLE for ENGLAND and WALES (Males) for 5-yearly periods by the same method previously described as used for the Haydock Life-Table, and from corresponding data, and the comparison of these results with those of the complete or extended Life-Table for England and Wales as given in the Registrar-General's Decennial Supplement for 1881-90.

(In the headings of the Table, "A" denotes the *extended* or yearly Life-Table, and "B" the *short* or 5-yearly Life-Table).

Age.	p_x			l_x			E_x		
	Mean chances of living one year in the interval following age x.			Number of Survivors at each age.			Mean after-lifetime, or Expectation of Life.		
x	A*	B	Differences of B from A.	A	B	Differences of B from A.	A	B	Differences of B from A.
0	—	—	—	509,180	509,180	—	43·66	43·51	— 0·15
5	·99516	·99467	—·00049	382,646	382,646	—	52·75	52·55	— 0·20
10	·99801	·99705	—·00096	373,472	372,551	— 951	49·00	48·91	— 0·09
15	·99622	·99545	—·00077	369,764	367,915	— 1,849	44·47	44·49	+ 0·02
20	·99468	·99457	—·00011	362,819	359,631	— 3,188	40·27	40·46	+ 0·19
25	·99283	·99299	+ ·00016	353,273	349,984	— 3,289	36·28	36·51	+ 0·23
30	·99099	·99144	+ ·00045	340,783	337,884	— 2,899	32·52	32·73	+ 0·21
35	·98890	·98870	—·00020	325,694	323,660	— 2,034	28·91	29·05	+ 0·14
40	·98624	·98650	+ ·00026	308,015	305,788	— 2,227	25·42	25·61	+ 0·19
45	·98284	·98282	—·00002	287,400	285,689	— 1,711	22·06	22·23	+ 0·17
50	·97793	·97844	+ ·00051	263,571	261,982	— 1,589	18·82	19·02	+ 0·20
55	·97040	·97026	—·00014	235,741	234,938	— 803	15·74	15·92	+ 0·18
60	·95861	·96050	+ ·00189	202,857	202,020	— 837	12·88	13·10	+ 0·22
65	·94155	·94334	+ ·00179	164,202	165,147	+ 945	10·31	10·47	+ 0·16
70	·91560	·91562	+ ·00002	121,507	123,368	+ 1,861	8·04	8·17	+ 0·13
75	·87741	·88101	+ ·00360	78,358	79,395	+ 1,037	6·10	6·31	+ 0·21
80	·82109	·82307	+ ·00198	40,746	42,141	+ 1,395	4·52	4·69	+ 0·17
85	—	—	—	15,207	15,917	+ 710	—	—	—

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*—These fractions represent the numerical values of logs. $\frac{p_x + p_{x+1} + p_{x+2} + p_{x+3} + p_{x+4}}{5}$ in each of the 5-yearly periods.

TABLE IX.

Comparing the results of two other shortened Life-Tables for England and Wales (Males), "C" and "D" (which have been calculated from the same data as the preceding Life-Table "B" in Table VIII.), with the results of the extended Life-Table "A," as regards the values of E_x (mean after-lifetime) obtained.

In all three Life-Tables, B, C, and D, the results of the complete English Life-Table A have been taken as already calculated for the first 5 years of life and from age 85 upwards.

"C" has been worked out by the short method of Dr. Farr.

"D" by a simple modification of Dr. Farr's method, as previously described.

MEAN AFTER-LIFETIME, OR EXPECTATION OF LIFE—E_x.						
Age.	C	Differences of C from A.	D	Differences of D from A.		
					Age.	
0	43·65	— ·01	43·42	— ·24		
5	52·74	— ·01	52·43	— ·32		
10	49·10	+ ·10	48·79	— ·21		
15	44·69	+ ·22	44·37	— ·10	20	+ ·15
25	36·71	+ ·43	36·38	+ ·10	30	+ ·20
35	29·28	+ ·37	28·92	+ ·01	40	+ ·19
45	22·48	+ ·42	22·09	+ ·03	50	+ ·26
55	16·22	+ ·48	15·77	+ ·03	60	+ ·40
65	10·88	+ ·57	10·33	+ ·02	70	+ ·59
75	6·91	+ ·81	6·23	+ ·13	80	+ ·75

In order to avoid the fallacy of generalising too much from one particular instance, I have, in addition, worked out the "short" method (by combining "C" and "D") as applied to the female section of the last Life-Table for England and Wales, and the results obtained are in accordance with those previously arrived at for the male section—in giving a very close approximation to the results of "A."

It has been found, however, that for age 10 the nearest approach to "A" is to be obtained by taking the arithmetical mean of the values of E_{10} by methods "C" and "D." The rules previously given are, therefore, to be modified to this extent.

Having submitted the results to Mr. A. C. Waters, they are considered by him to give "*a remarkable approximation to the exact values.*"

He has also made to me the suggestion, which would not otherwise have occurred to me, that something further is to be got out of method "D"; that is, by taking the values of E_x obtained for the ages 15, 25, 35, 45, etc , and applying to them the formulæ for interpolation (given on page 20), it may be possible to obtain values of E_x for the even ages, 20, 30, 40, etc., approximating to the true values deduced by "A."

Acting upon this suggestion, I have worked out the results which (as set down in Table X.) are surprisingly accurate.

The formulæ have been applied both to the numbers themselves, and to the logarithms of the numbers, and as, on the whole, the results obtained from the *simple numbers* give the nearest approach to the exact values (although there was found to be but little difference), this method has been chosen as being at once *the most simple and the most accurate.*

The mode, therefore, of obtaining the value of E_{20} (that is, the expectation of Life at age 20) is simply this:—

Having given the values of E_{5}, E_{15}, E_{25}, and E_{35}, by the formula, the "CENTRE term" is thus deduced:—

$$E_{20} = \frac{10\ (E_{15} + E_{25}) - (E_{5} + E_{15} + E_{25} + E_{35})}{16}$$

that is, from 10 times the sum of the middle terms subtract the sum of all four terms and divide the remainder by 16.

Thus, in the MALE section of the table:—

$$E_{20} = \frac{10\ (44{\cdot}37 + 36{\cdot}38) - (52{\cdot}74 + 44{\cdot}37 + 36{\cdot}38 + 28{\cdot}92)}{16} = 40{\cdot}32$$

In a similar way the values of E_{30}, E_{40}, E_{50}, and E_{60} have been calculated.

E_{70} has been arrived at by the formula:—

$$E_{70} = \frac{E_{75} + E_{55}}{4} + 1\tfrac{1}{2}\ E_{65} - E_{60}$$

Therefore it would appear that the most simple and the most accurate way of calculating a local Life-Table (IN SO FAR AS THE E_x COLUMN IS CONCERNED) would be to calculate the mean values of p_x (the chance of living one year from age x to age $x + 10$) *directly* from the true mean population numbers and numbers of deaths for the 10-yearly periods 5-15,* 15-25, etc., and then having arrived at the values of E_{5}, E_{15}, E_{25}, etc., by methods "C" and "D," to "interpolate" from these values the values of E_{10}, E_{20}, E_{30}, etc.

The difference of this method from the method adopted for the Haydock Life-Table is that the "interpolation" is left until the *last* stage, instead of being effected at the *first* stage, in the numbers of population and of deaths. The "interpolations" in the E_x column can be calculated in a very short time on a small sheet of notepaper, whereas the calculations as applied to the numbers of population and deaths are rather long and tedious processes.

Still, if it be desired to get intermediate quinquennial values in the "p_x" and "l_x" columns of the Life-Table, which, as has been explained, HAVE THEIR OWN IMPORTANCE as well as the "E_x" column, it will be necessary to make the interpolations *at the beginning.*

In order to save space and the expense of printing I have not given the five Life-Tables in full, as they have been worked out by methods "B," "C," and "D."

It would scarcely require the methods of the "higher criticism" to point out how many different "strata" exist in the preceding pages, and what apparent inconsistencies are to be found. The only apology or explanation which can be offered for such, is, that a process of "evolution" has been represented.

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*—In the Tables which I have worked out, the 5-yearly periods, 5-10 and 10-15 have been dealt with separately.

TABLE X.

Values of E_x for ENGLAND AND WALES (1881-90) calculated by the combined short methods "C" and "D," compared with "A."

For Ages 0 and 5 values of "C" are taken.
For Age 10 MEAN values of "C" and "D" are taken.
For Ages 15, 25, 35, &c., values of "D" are taken.
For Ages 20, 30, 40, &c., the values are "interpolated" from the values of 15, 25, 35, &c., already found by "D."

Ages.	MALES		FEMALES.	
	E_x	Differences from "A."	E_x	Differences from "A."
0	43·65	— ·01	47·25	+ ·07
5	52·74	— ·01	55·01	+ ·09
10	48·95	— ·05	51·13	+ ·03
15	44·37	— ·10	46·60	+ ·05
20*	40·32	+ ·05	42·52	+ ·10
25	36·38	+ ·10	38·56	+ ·06
30	32·58	+ ·06	34·78	+ ·02
35	28·92	+ ·01	31·13	— ·03
40	25·43	+ ·01	27·56	— ·04
45	22·09	+ ·03	24·06	+ ·01
50	18·84	+ ·02	20·55	— ·01
55	15·77	+ ·03	17·19	— ·04
60	12·91	+ ·03	14·08	— ·02
65	10·33	+ ·02	11·26	± ·00
70	8·09	+ ·05	8·80	+ ·03
75	6·23	+ ·13	6·77	+ ·09

*—These values of E_{20} have been worked out by the formula given on the preceding page. However, as E_5 does not belong to the same series, viz., the values deduced by method "D," I have found that the following formula gives still truer results:—

$$E_{20} = \frac{E_{15} + E_{35}}{4} + 1\tfrac{1}{2} E_{25} - E_{30}$$

This gives the value of E_{20} for *Males* 40·31, with a difference from "A" of + ·04, and for *Females* 42·49, with a difference from "A" of + ·07.

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